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Bounded Rationality in a Cournot Duopoly Game

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Abstract

This paper analyzes choices and welfare in a Cournot duopoly setting with linear demand using four models of bounded rationality. The models studied in this paper are Level-k, Cognitive Hierarchy, Asymmetric Quantal Response and Noisy Introspection. It is found that in the Level-k model choices, profits and welfare alternate around the Nash Equilibrium levels depending on whether the level is odd or even. In the Cognitive Hierarchy model the choices of the first two types (L-0 and L-1) coincide with the choices in the Level-k model, a L-2 produces a smaller quantity while the quantity of a L-3 is higher or lower depending on the value of a particular parameter in the model. Both in the Asymmetric Quantal Response and Noisy Introspection models we find that choices are spread around the Nash Equilibrium level for all parameter values and thus welfare is below the Nash Equilibrium benchmark. We also use parameter estimates from other well-known experiments to obtain an approximation to empirically plausible welfare levels.

JEL Classification**:** C72; D21.
Keywords**:** Cournot Game; Bounded Rationality; Level-k Model; Asymmetric Quantal Response Equilibrium, Noisy Introspection; Cognitive Hierarchy.

Resumen

Este artículo analiza las elecciones de cantidad y los efectos sobre el bienestar en un modelo de duopolio de Cournot con demanda lineal. Los modelos que se estudian en este trabajo son los de racionalidad acotada Nivel-k, Jerarquía Cognitiva, Respuesta Quantal Asimétrica e Introspección Imprecisa. Se encuentra que en el modelo de Nivel-k las elecciones de cantidad y los niveles de bienestar alternan alrededor del nivel de equilibrio de Nash, dependiendo de si los niveles son pares o impares. En el modelo de Jerarquía Cognitiva, las

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elecciones de los primeros dos niveles (Nivel 0 y 1) coinciden con las elecciones del modelo Nivel-k, mientras que los niveles 2 y 3 difieren. Los restantes dos modelos generan elecciones de cantidad alrededor del equilibrio de Nash y, por lo tanto, niveles de bienestar inferiores. Finalmente, se usan estimaciones experimentales de parámetros para obtener aproximaciones a los niveles de bienestar empíricamente validas.

***Clasificacion JEL*:** C72; D21.
***Palabras Clave*:** juego de Cournot; racionalidad acotada; modelo de nivel-k, equilibrio de respuesta cuantil asimétrico; jerarquía cognitiva, introspección imprecisa.

Introduction

This paper analyzes a standard Cournot duopoly game with linear demand using several well-known non-equilibrium models of bounded rationality (Level-k, Cognitive Hierarchy, Noisy Introspection and Asymmetric Quantal Response). The motivation for the analysis comes from the fact that equilibrium concepts correctly describe behavior when agents participate in a game repeatedly, thus having time to learn. However, there are many situations where the possibility of learning is indeed quite limited. For example, if the environment where managers operate changes due to variations in cost or demand we should expect convergence to equilibrium to be slow or impossible. Moreover, Cournot competition can be interpreted as a choice of capacity (Kreps and Scheinkman, 1983) followed by price competition, and since the investment in capacity is generally a sunk cost we should expect constraints in the ability of firms to adjust[1]. The four models analyzed here include Nash Equilibrium as a particular case, thus they will always describe behavior better than it. The question of which one describes choices more realistically is an empirical one, which will not be attempted here. Nevertheless, it is interesting to investigate what theoretical results can be derived from these models of bounded rationality and what are their implications.

There are several reasons why subjects may fail to choose the Nash Equilibrium strategies. This concept requires the mutual consistency of actions and beliefs, that is, what one player chooses must be optimal given her beliefs about the choice of the other player. And conversely, what a player thinks the

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[1] There exists vast evidence showing that experimental subjects, typically undergraduate students, deviate from the Nash Equilibrium prediction in simple Cournot games. See Camerer (2003) and Kagel and Roth (1995) for a comprehensive survey of experimental results.

other will do must be correct given her own action. The Nash strategies are a fixed point where all agents have correct beliefs and no one wishes to deviate. However, the cognitive requirements of this concept are certainly quite high. People in general have problems with higher order reasoning (I think that you think that I think ...), thus complicating the thought process necessary to reach equilibrium. But even if an individual is rational Nash Equilibrium also demands that each player believes her rival is equally rational (or has the same cognitive skills), however, there is considerable evidence showing that most people are overconfident with respect to their relative intelligence. The models analyzed in this paper (Level-k, Cognitive Hierarchy, Noisy Introspection and Asymmetric Quantal Response) describe non-equilibrium concepts in which actions and beliefs are not consistent. More precisely, the former are optimal (or near optimal) given the latter but the converse is not true.

We focus on these non-equilibrium models because we are interested in analyzing the behavior of firms before they have time to learn. Moreover, the Cournot model considered here can be solved by deletion of dominated strategies. This naturally leads to consider solution concepts with different types or levels of reasoning, with higher levels performing more iterations than lower ones. Both the Level-k and Cognitive Hierarchy models have this characteristic, however, they lack sensitivity of actions to payoffs. In order to see how this sensitivity affects choices we also consider Noisy Introspection and Asymmetric Quantal Response, where quantities leading to higher payoffs are chosen with higher probability.

We find that in the Level-k model quantities, profits and welfare oscillate around the Nash Equilibrium depending on the type of firms. In the Cognitive Hierarchy model the actions of the first two types of players (L-0 and L-1) are the same as in the Level-k model. However, for L-2 the quantity is smaller while a L-3 player will choose a smaller or larger quantity depending on the value of the model parameter describing beliefs towards higher order types. In both Asymmetric Quantal Response and Noisy Introspection, for all parameter values, choices are spread around the Nash Equilibrium quantity and expected welfare is below the Nash Equilibrium level.

The aforementioned models have been applied in the last couple of decades to describe choices in experimental settings. For example, just to name a few, Bosch-Domenech, Montalvo, Nagel, and Satorra (2002) and Nagel (1995) used the Level-k model to describe choices in Beauty Contests while Crawford and Iriberri (2007) applied it to the analysis of Auctions. Noisy Introspection was used by Goeree and Holt (2004) to study choices of a set of experiments ranging from several variations of the asymmetric prisoner’s dilemma to coordination games. Camerer, Ho, and Chong (2004) introduced the Cognitive Hierarchy concept and estimated a large number of experiments with it.

Gneezy (2005) found that the Cognitive Hierarchy model successfully predicted bids in common value second price auctions. Weizsacker (2003) has shown that Asymmetric Quantal Response describes choices in several one-shot games better than the standard symmetric Quantal Response Equilibrium.

2. Models

In this section we will describe the four models of bounded rationality used to analyze the Cournot game. The game is a standard duopoly interaction with linear demand and constant marginal cost where both firms choose quantity simultaneously. Market demand is given by $P = a - b(q_A + q_B)$ where q_i is the quantity chosen by firm i and costs are given by $C(q_i) = cq_i$, we assume the standard $a > b > 0$ and $a > c$. Both firms choose quantity simultaneously. As is well-known the unique Nash equilibrium is $q^* = \frac{a-c}{3b}$ with profits $\frac{(a-c)^2}{9b}$ and welfare $\frac{4(a-c)^2}{9b}$. Below in the following subsections we will introduce each model of bounded rationality and apply it to the Cournot game described above.

2.1. Level-k

In a Level-k model (Nagel, 1995; Stahl and Wilson, 1995) there are different types[2] corresponding to different depths of reasoning. The lowest level (Level-0) does not comprehend the situation well and chooses randomly on some interval, thus a L-0 player does not best respond to any belief. A Level-1 player believes that the other is L-0 and chooses the quantity to maximize expected profits. A Level-2 player thinks the other is L-1 and so on, in general, a L-k player thinks the other is L-(k-1) and best responds to that belief. Notice that this is not an equilibrium model because actions are consistent with beliefs but beliefs inconsistent with actions. In addition, beliefs are degenerate in the sense that a Level-k player thinks the other is one level below with probability one. It may be argued that this is quite an extreme assumption, but one that simplify the calculations of the optimal choices considerably. A different model we will analyze (Cognitive Hierarchy) relaxes this assumption to allow for non-degenerate beliefs. Usually, it is assumed that a L-0 player chooses randomly following a uniform distribution and since no firm will choose a quantity that drives price below marginal cost (assuming the other firm produces nothing), then a L-0 will choose uniformly in the interval $[0, \frac{a-c}{b}]$.

The choice of all other positive levels can be found iteratively as a best

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[2] In this paper we will use the terms level, type and depth interchangeably.

response to the previous level. Notice that this is equivalent to find the strategies that survive the iterative elimination of dominated strategies starting from a uniform prior. The proposition below shows the quantity produced by a Level-k firm.

Proposition 1. *A Level-k firm for* $k \geq 1$ *chooses quantity* $m_k\left(\frac{a-c}{b}\right)$ *with* $1 > m_k = \frac{1}{3}\left(\frac{-1^k+2^{k+1}}{2^{k+1}}\right) > 0$.

Proof. The optimization problem for firm i is $\max_{q_i \geq 0}(a - b(q_i + q_j) - c)q_i$.

The profit function is strictly concave due to the linearity of the demand function. The interior solution to this problem is $q_i = \frac{a-bq_j-c}{2b}$. Since a Level-k firm believes the other is Level-$(k-1)$ then we have $q_{i,k} = \frac{a-bq_{j,(k-1)}-c}{2b}$. It is straightforward to check that replacing q_k with $m_k\left(\frac{a-c}{b}\right)$ satisfies the equality.

Q.E.D.

Since $1 > m_k > 0$ the production level lies strictly in the interval $(0, \frac{a-c}{b})$. Also it is true that as $k \to \infty$, q_k converges to the Nash Equilibrium quantity $q_{NE} = \frac{a-c}{3b}$. The sequence $\{q_k\}_{k\geq 1}$ oscillates around the Nash Equilibrium level with a very quick convergence due to both numerator and denominator growing very fast, at the rate of 2^{k+1}. More specifically, the ratio q_k/q_{NE} for the first six levels is given by the values $0.75, 1.125, 0.937, 1.031, 0.984, 1.008$. Finally, it is the case that $q_k < q_{NE}$ for k odd and $q_k > q_{NE}$ for k even.

The issue of profits in this model is more delicate since firm's i profits depend on its level and the level of the competitor, thus many possible combinations are possible. Two important cases are when the firm correctly guesses the level of the rival (for example if firm i is Level-2 then the other firm *actually* is Level-1) and the other is when both firms are of the same level (notice that in this case the production level of the rival firm is incorrectly guessed). We can define Π_k^c to be the profit level of a Level-k firm when it correctly guesses the level of the rival and Π_k^i the profit of a Level-k firm when both have the same level, thus incorrectly guessing the level of the rival. Replacing the corresponding quantities in both profit functions and simplifying yields

$$\Pi_k^c = \frac{((-1)^k + 2^{1+k})^2(a-c)^2}{2^{2(1+k)}9b}$$

$$\Pi_k^i = \frac{(-(-1)^k + 2^k)((-1)^k + 2^{1+k})(a-c)^2}{2^{2k+1}9b}$$

Notice that $\lim_{k\to\infty}\Pi_k^c = \lim_{k\to\infty}\Pi_k^i = \frac{(a-c)^2}{9b} = \Pi_N$. Profits converge in both cases to the Nash Equilibrium level. How do profits compare to the Nash Equilibrium benchmark for finite k? The next proposition answers the question.

Proposition 2 *Let Π_k^c and Π_k^i be the profit level when the Level-k firm correctly and incorrectly predicts the level of the rival, respectively. Then for $k \geq 1$ we have $\Pi_k^c < \Pi_N$ for k odd and $\Pi_k^c > \Pi_N$ for k even. And also $\Pi_k^i > \Pi_N$ for k odd and $\Pi_k^i < \Pi_N$ for k even.*

Proof. The difference in profits when the firm correctly guesses is given by

$$\Pi_k^c - \Pi_N = \frac{\left((-1)^k+2^{1+k}\right)^2(a-c)^2}{2^{2(1+k)}9b} - \frac{(a-c)^2}{9b} = \left(\frac{\left((-1)^k+2^{1+k}\right)^2}{2^{2(1+k)}} - 1\right)\frac{(a-c)^2}{9b}.$$

The expression in parenthesis determines the sign of $\Pi_k^c - \Pi_N$ since $\frac{(a-c)^2}{9b} > 0$. Clearly $(-1)^k < 0$ for k odd and $(-1)^k > 0$ for k even. Adding 2^{k+1} on both sides and squaring we get $((-1)^k + 2^{k+1})^2 < 2^{2(k+1)}$ for k odd and $>$ for k even. Dividing both sides by $2^{2(k+1)}$ we get the desired result.

In the second case the difference in profits is given by

$$\Pi_k^i - \Pi_N = \frac{(-(-1)^k + 2^k)((-1)^k + 2^{1+k})(a-c)^2}{2^{2k+1}9b} - \frac{(a-c)^2}{9b}$$
$$= \left(\frac{\left(-(-1)^k+2^k\right)\left((-1)^k+2^{1+k}\right)}{2^{2k+1}} - 1\right)\frac{(a-c)^2}{9b}.$$

Like in the first case the sign of $\Pi_k^i - \Pi_N$ depends on the sign of the expression in parenthesis. Notice that $0 < \frac{1}{2^k} < 1$ for $k \geq 1$, thus $-(-1)^k > \frac{1}{2^k}$ for k odd and $<$ for k even. Rearranging gives $-1 - (-1)^k(2^k) = -1 + (-1)^k(-2^{k+1} + 2^k) > 0$ for k odd and $<$ for k

even. Also notice that $-1 = -(-1^k)(-1^k)$. Also adding 2^{2k+1} on both sides gives $-(-1^k)(-1^k) - (-1)^k 2^{k+1} + (-1)^k 2^k + 2^{2k+1} > 2^{2k+1}$ for k odd and $<$ for k even. Dividing both sides by 2^{2k+1} and factoring gives the desired inequality.

Q.E.D.

It is straightforward to compute the welfare when both firms are Level-k. Since welfare is given by

$$W_k = \int_0^{Q_k} (a - bt)dt - 2\int_0^{\frac{Q_k}{2}} cdt = aQ_k - \frac{b}{2}Q_k^2 - cQ_k$$

where Q_k is the total production of Level-k firms. Evaluating the expression above yields

$$W_k = \frac{((-1)^k + 2^{1+k})(-(-1)^k + 2^{2+k})(a-c)^2}{2^{2k+1}9b}$$

which is smaller than the welfare level of the Nash Equilibrium for k odd and larger for k even and converges to $\frac{4(a-c)^2}{9b}$ when $k \to \infty$.

2.2. Cognitive Hierarchy

The next model we analyze is Cognitive Hierarchy of Camerer *et al.* (2004). It is similar to the Level-k model in the sense of consisting of different types of players. A Level-0 player chooses randomly in an interval. A Level-1 player thinks the other is L-0 and maximizes expected profits given this belief. A Level-2 player, unlike the standard Level-k model, believes the other player can be either L-0 or L-1 with positive probability. In general, beliefs for levels higher than one are non-degenerate. The appealing feature of this model is that the sequence of beliefs can be determined with a single parameter τ. More precisely, a L-k player believes that she is facing a $L-j,\ \ j<k$ with probability $\frac{f(j|\tau)}{\sum_{l=0}^{j} f(l|\tau)}$ where $f(j|\tau)$ is the probability density function of the Poisson distribution with $\tau > 0$. The parameter τ measures the bias towards higher levels in beliefs, actually, the Level-k model is a special case of this one as $\tau \to \infty$. Thus, given τ, the beliefs of a Level-k player can be constructed iteratively starting from the probability choice vector of a level zero player $P0(q)$, then computing the vector of a L-k with

$$Pk(q_i) = \sum_{m=0}^{k-1} \frac{f(m|\tau)}{\sum_{l=0}^{k-1} f(l|\tau)} Pm(q_i).$$

This model also captures the intuitive property that the marginal benefit of thinking harder (and thus increasing depth of reasoning) is decreasing. Applied to the Cournot game a Level-0 player chooses randomly between $[0, \frac{(a-c)}{b}]$ and a Level-1 player $\frac{1}{4}\frac{(a-c)}{b}$. Since $f(k|\tau) = \frac{\tau^k e^\tau}{k!}$, a Level-2 player thinks the other is L-0 with probability $\frac{f(0|\tau)}{f(0|\tau)+f(1|\tau)} = \frac{1}{1+\tau}$ and L-1 with probability $\frac{f(1|\tau)}{f(0|\tau)+f(1|\tau)} = \frac{\tau}{1+\tau}$. A Level-2 chooses the quantity that maximizes expected profits given those beliefs

$$\max_{q_i} E_2[(P(q_i + q_{-i}) - c)q_i] = (a - b(q_i + E_2[q_{-i}]) - c)q_i$$

where $E_2[q_{-i}] = \frac{1}{1+\tau}\frac{(a-c)}{2b} + \frac{\tau}{1+\tau}\frac{(a-c)}{4b}$ is the L-2 expected quantity given τ.

The solution to the optimization problem is given by $q_i(\tau) = \frac{(a-c)(2+3\tau)}{8b(1+\tau)}$.

Once the optimal choice of a L-2 player is obtained the expected profit of a L-3 is straightforward to compute. The problem for a L-3 is

$$\max_{q_i}(a - b(q_i + E_3[q_{-i}] - c)q_i$$

where

$$E_3[q_{-i}] = \frac{1}{1+\tau+\tau^2/2}\frac{(a-c)}{2b} + \frac{\tau}{1+\tau+\tau^2/2}\frac{(a-c)}{4b} + \frac{(a-c)(2+3\tau)}{8b(1+\tau)}\frac{\tau^2/2}{1+\tau+\tau^2/2}$$

A Level-3 player will choose $q_i(\tau) = \frac{a-bE_3[q_{-i}]-c}{2b}$ to maximize expected profits.

Simplifying the expression above yields the optimal choice of a Level-3 player

$$q_i(\tau) = \frac{(a-c)(8+20\tau+18\tau^2+5\tau^3)}{16b(1+\tau)(2+2\tau+\tau^2)}$$

In general, the choice of a Level-k player can be constructed iteratively from lower levels. However, the complexity of the solution grows very fast, therefore we have decided to analyze the effect of τ on the optimal[3] quantity up to Level-3.

Given the similarity of the Level-k and Cognitive Hierarchy models it is interesting to analyze in what aspects they differ. By definition, both L-0 and L-1 choose the same quantities in both models. It is easy to see that for a Level-2 firm the Level-k model predicts a quantity larger than in the CH model since $\frac{3}{8}(a-c)/b > \frac{(2+3\tau)}{1+\tau}\frac{(a-c)}{8b}$ for all $\tau > 0$. A L-2 firm in the Level-k model best responds with a large quantity to a relatively low production level[4] by a L-1 firm given that in the Cournot game reaction functions are downward sloping. However, a L-2 firm in the CH model thinks for all finite values of τ that it is facing both levels with positive probability, the expected production level of rivals is higher and thus the best response involves a smaller quantity. A L-3 player in the Level-k model produces $\frac{3}{8}\frac{(a-c)}{b}$ which is larger than the quantity produced by a L-3 in the CH model if $\tau < \surd(2/3)$ and smaller otherwise. Here a L-3 player in the CH model will best respond to a high quantity when τ is low (because she believes the other is L-0 with a relatively high probability), therefore the optimal quantity will be small. On the other hand, when τ is large the belief is the opposite, the firm thinks the rival will choose a small quantity thus best responding with a larger quantity.

In terms of welfare this model is similar to the Level-k (exactly the same for L-0 and L-1), with the difference that a L-2 will choose a lower quantity with the consequent lower welfare level and that a L-3 will choose a higher or lower quantity depending on the value of the parameter with the corresponding higher or lower welfare level.

2.3. Asymmetric Quantal Response

Next we analyze the Asymmetric Quantal Response model of Weizsacker (2003). This model is a non-equilibrium version of the Quantal Response Equilibrium (McKelvey and Palfrey, 1995). Since AQR is a derivation of QRE we first describe the latter. Quantal Response Equilibrium is a generalization

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[3] Analyzing only up to L-3 can also be justified on the grounds that subjects would hardly take the trouble of making the computations of higher levels if the marginal benefit of doing so is small.

[4] Relative to the Nash Equilibrium quantity $\frac{(a-c)}{3b}$.

of Nash Equilibrium in the sense that players "better" respond, instead of best responding, given their beliefs about the actions of other players. Actions generating higher expected profits are chosen with higher probability, thus allowing for mistakes at the time of selecting actions. Usually the link between expected profits and probabilities is given by a logit function with parameter[5] μ

$$P(q_i) = \frac{\exp\{E_{P'}\Pi(q_i)/\mu\}}{\sum_i \exp\{E_{P'}\Pi(q_i)/\mu\}} \tag{1}$$

where in our analysis of the Cournot game

$$E_{P'}[\Pi(q_i)] = \sum_j P'(q_j)(a - b(q_i + q_j) - c)q_i = aq_i - bq_i^2 - cq_i - bq_i \sum_j P'(q_j)q_j \tag{2}$$

is the expected profit for firm i of choosing q_i when beliefs about the choice of firm j are P'. The parameter μ measures the inverse of the sensitivity of actions to expected payoffs. A larger μ implies a less sensitive player, in the limit, as μ goes to infinity the choice probability vector approaches a uniform distribution. On the other hand, as μ decreases the sensitivity increases, making the probability vector more concentrated around the Nash Equilibrium. In the limit as it goes to zero the probability vector becomes degenerate. Quantal Response Equilibrium is an equilibrium concept and as such there is consistency between actions and beliefs, that is $P = P'$ in equilibrium.

However, in this paper we are interested in the learning process leading to equilibrium, thus we analyze the Asymmetric Quantal Response model. In this version, player i believes that her rival j is less responsive to payoffs (this may occur as a consequence of overconfidence about her own intelligence or ability), that is $\mu_i < \mu_j$, thus leading to a belief vector P not in equilibrium. Formally,

$$P(q_i) = \frac{\exp\{E_{P'}\Pi(q_i)/\mu_i\}}{\sum_i \exp\{E_{P'}\Pi(q_i)/\mu_i\}} \tag{3}$$

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[5] There are other ways to describe this relationship, for example with a power function.

$$P'(q_j) = \frac{\exp\{E_P \Pi(q_j)/\mu_j\}}{\sum_j \exp\{E_P \Pi(q_j)/\mu_j\}} \quad (4)$$

The vector of actual choices $P(q_i)$ will be different to the vector of beliefs $P'(q_j)$ as long as $\mu_i \neq \mu_j$. Since the solution cannot be derived in close form we computed it for the parameter values $a = 100, b = c = 1$. Figure 1 shows the vector $P(q_i)$ for three different values of μ_i. Notice how the distribution becomes more spread around the Nash Equilibrium quantity as μ_i increases. Once the choice vector is computed, it is straightforward to calculate expected welfare $E_{P(q_i)}[W(q_i)] = aE_{P(q_i)}(Q) - \frac{b}{2}E_{P(q_i)}[Q^2] - cE_{P(q_i)}[Q]$.

Figure 1

AQR Probability Vector for Different Values of μ

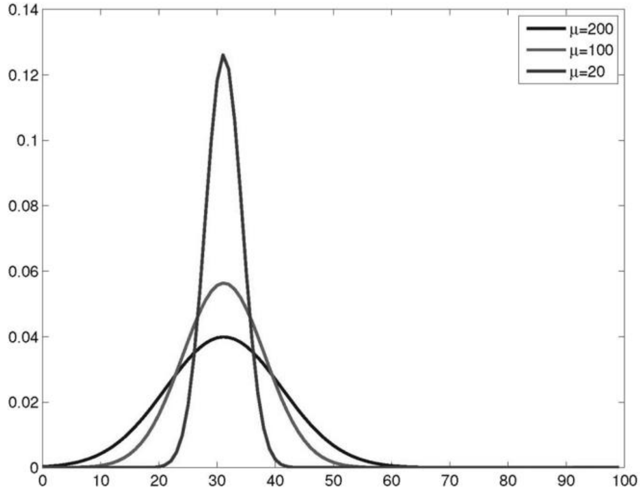
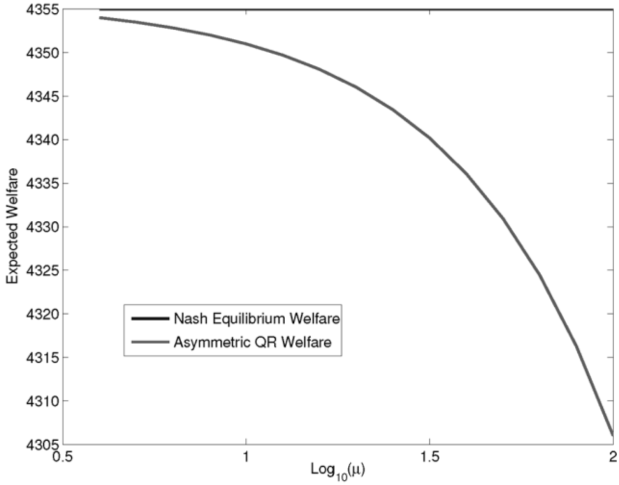


Figure 2 shows that expected welfare is a decreasing function of μ_i, and as expected, as the sensitivity parameter approaches zero $E[W(q)_i]$ gets closer to the welfare level of the Nash Equilibrium. This occurs because expected welfare is a strictly concave function of quantity and the equilibrium vector $P(q_i)$ is symmetrically spread around the Nash Equilibrium. Thus by Jensen Inequality $W(q^{NE}) = W(E_P[q]) > E_P[W(q)]$.

Figure 2
Asymmetric Quantal Response ($\mu_j = 100$) and Nash Equilibrium Welfare Levels



2.4. Noisy Introspection

Noisy Introspection (Goeree and Holt, 2004) is a non-equilibrium model that generalizes the concept of rationalizability. It consists of layers of beliefs where the action of a player depends on what she thinks the other player will do (that is her first order belief). However, if this is not an equilibrium concept and beliefs need not be consistent with actions, then how are first order beliefs determined? They depend on what she thinks about what the other player thinks (that is her second order belief). Her second order belief depends on the third order and so on. We will follow Goeree and Holt (2004) and model the link between layers with the logit function

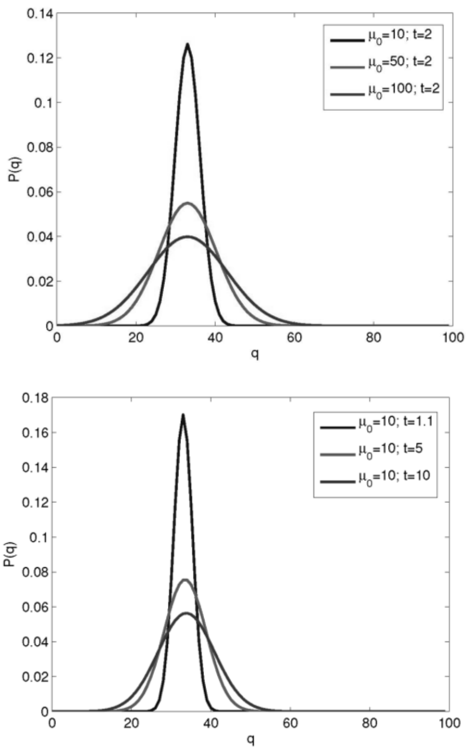
$$PN_{\mu_N}(q_i) = \frac{\exp\{E_{P(N+1)}\Pi(q_i)1/\mu_N\}}{\sum_i \exp\{E_{P(N+1)}\Pi(q_i)1/\mu_N\}} \quad (5)$$

Equation 5 determines the sequence of beliefs PN for $N \in \{0,1,2,\dots\}$. The vector $P0$ represents the zero-th order belief (The actual choice of players) and PN the N-th order belief. The actual probability choice vector can be obtained starting from any sufficiently large initial order belief and iterating on equation 5. It is also natural to assume that beliefs become more imprecise with higher orders, that is, $\mu_N \geq \mu_{N-1} \geq \ldots \geq \mu_0$. However, a more parsimonious specification depending only on two parameters can be obtained by assuming that the sensitivity parameters in the logit equation are given by $\mu_k = t^k \mu_0$ for $k \in 1,2,\ldots$ and $t > 1$. The restriction $t > 1$ arises from the fact that if $t = 1$ the NI vector $P0$ is equivalent to the equilibrium vector of a Quantal Response Equilibrium.

Figure 3 shows the probabilities of each action for several values of t and μ_0. As both μ_0 and t increase the vector becomes more spread around the Nash Equilibrium. Expected welfare in this model is a decreasing function of both parameters and smaller than the Nash Equilibrium level. The reason for this is the same as in QRE, spreading choices around the Nash Equilibrium leads to a lower expected welfare due to the strict concavity of the function.

Figure 3

Left: NI Probability Vector for Different Values of μ_0. Right: NI Probability Vector for Different Values of t



Conclusion

In this paper we have analyzed the effect of bounded rationality on choices in a standard Cournot duopoly model. We have found that in a Level-k model the effect depends on the level of firms. Choices and welfare alternate from lower to higher than the Nash benchmark depending on whether types are odd or even, but quickly converge to the Nash level. The quantities and welfare in the Cognitive Hierarchy model are equal to those derived in the Level-k model for the first two levels. However, Level-2 firms choose a lower quantity in this model relative to the Level-k for all parameter values. Level-3 firms choose a lower quantity (again relative to Level-k) if $\tau < \sqrt{\frac{2}{3}}$ and larger otherwise. However, both in Asymmetric Quantal Response and Noisy Introspection welfare is found to be lower than the Nash level since choices are spread around the Nash Equilibrium quantity.

In order to get a better appreciation of how the models compare we may use parameter estimates from other experiments. In particular, for the Level-k and Cognitive Hierarchy we use estimates from a well-known dominant solvable game like the Beauty Contest. The proportion of Level-k players is taken from the analysis in Bosch-Domenech *et al*. (2010) and the estimate of τ in the CH model from Camerer *et al*. (2004). Both estimates come from three well-known large-scale newspaper experiments. For Noisy Introspection we use estimates from Goeree and Holt (2004) for a broad set of games. In order to make the comparison more transparent we use the parameterization $a = 100$ and $b = c = 1$.

Table 1 shows the estimates and expected welfare in each case. In the Level-k model expected welfare is higher than the Nash Equilibrium level while in CH and NI it is lower[6]. The difference in results arises from the fact that in the Level-k model the proportion of even levels is relatively high. With our parameterization the NE is 33 while a L-0 chooses randomly in the interval [0,99] (with an expected value of 49.5), L-1 chooses 24.75 and L-2 chooses 37.125. Thus by giving a large weight to L-0 and L-2 expected quantities are higher than the NE level. On the other hand, the $\tau = 3$ of Cognitive Hierarchy generates the proportions 0.077, 0.231, 0.346 and 0.346 for L-0 to L-3 respectively. Notice here the higher proportion of odd levels (choosing lower quantities than NE). And lastly, and not surprisingly given the previous section, NI generates lower expected welfare, however, it is interesting to see that the difference with NE is actually smaller than CH.

[6] We have not used the estimates in Weizsacker (2003) for the AQR because with them the Cournot model fails to converge. However, from a qualitatively standpoint we know that expected welfare will be lower than the NE level.

Table 1
Estimates and Expected Welfare

Model	**Estimates**	**Expected Welfare**
Nash Eq.	–	4356
Level-k	$\pi_0 = 0.3$; $\pi_1 = 0.09$; $\pi_2 = 0.22$; $\pi_\infty = 0.39$	4464.6
CH	$\tau = 3$	4267.6
NI	$\mu = 4.4$; $t = 4.1$	4342.3

What model describes choices more accurately with experimental data? We do not have an answer to this question yet. Nevertheless, for future work we plan to use data from a Cournot experiment to estimate the four models and determine which one provides the best fit.

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