En this article, we present an evolutionary model that shows the possibility of political cycles in which populist and non-populist elites alternate in power. The platforms promoted by populist politicians are presented as alternatives to solve pressing problems that afflict society, without paying attention to the economic or social imbalances that they can generate, while the policies promoted by a non-populist government do not usually consider the difficulties that long-term growth policies can generate in some sectors of society. In this work we will show that this behavior can give rise to the emergence of cycles of alternating power given to populist and non-populist rulers depending on the urgencies of society. Furthermore, we will analyze the conditions under which society evolves towards a populist government with growing citizen approval, as well as the possibility of achieving a non-populist government with growing citizen support.

Palabras clave: Populismo, políticas de crecimiento, ciclos políticos.

pueden generar, mientras que las políticas impulsadas por un gobierno no populista no suelen considerar las dificultades que desde hace mucho tiempo las políticas de crecimiento a largo plazo pueden generar en algunos sectores de la sociedad. En este trabajo mostraremos que este comportamiento puede dar lugar al surgimiento de ciclos de alternancia de poder entre gobernantes populistas y no populistas en función de las urgencias de la sociedad. Además, analizaremos las condiciones en las que la sociedad evoluciona hacia un gobierno populista con creciente aprobación ciudadana, así como la posibilidad de lograr un gobierno no populista con creciente apoyo ciudadano.

Introducción

Nowadays we are witnessing a shift from traditional politics based on class and the left-right divide to a distinction based on cultural attitudes and education. This change is having a profound effect on the political systems of democracies organized according to the traditional left-right division. The traditional economic and redistributive conflict between the left and right is fading. Instead, a new conflict has emerged between nationalist and socially conservative positions versus cosmopolitan and socially progressive positions. Although it is possible to distinguish the different variants between the antagonists, we will not go into more detail at this time. We will show this new political division between the populists and non-populists. Without a doubt a somewhat schematic distinction but is appropriate for the purposes of this work. We refer the interested reader to Bonomi, Gennaioli and Tabellini (2021) and Rodrik (2017)).

In support of our schematic classification, we can resort to authors such as, Ware (2002) defines populism as a “political strategy deployed by a wide range of politicians”, while Mudde (2004) argues that populism is “an ideology that considers society to be ultimately separated into two homogeneous and antagonistic groups, “the pure people” versus “the corrupt elite” and which argues that politics should be an expression of the volonté générale (general will) of the people.” According to Barr (2009) for example, populism is “a mass movement led by an outsider or maverick seeking to gain or maintain power by using anti-establishment appeals and plebiscitarian linkages”.

Unlike Bischi, Favaretto and Carrera (2020), our work endorses the conclusions of Dornbusch and Edwards (2007). This work points out the existence of cycles of populism, guided by a policy that begins with providing income support to the neediest classes through public spending. They implement it through social programs that are actually an
instrument to attract clientele bases. This need is increasingly growing, causing greater fiscal deterioration and distortions in spending items. For this reason, fiscal discipline takes a back seat, since the well-being of the population is supposedly considered more important than healthy public finances or paying attention to the inflation that is inevitably generated, for further information see Rubli: (2023). However, on the other side, it is necessary to highlight that the globalization of markets, the growing development of technology, and free competition sponsored by non-populist or liberal policies, can give rise to a lack of satisfaction in social sectors that, even when initially supported by political platforms. In this sense, they feel disappointed in their aspirations and end up preferring short-term policies generally promoted by populist sectors. An adequate compensation policy could avoid the cycle of populism. However, it does not seem to be present in non-populist platforms.

The possibility of populism-non-populism cycles that our model explains opens the possibility of considering compensatory policies to avoid said cycle and, consequently, the economic deterioration that populist policies give rise to in the long term, see section 5.2.

Populism appears in many different forms in political life; there is a clear distinction between what we can call left-wing populism and right-wing populism, Krämer (2017). The first form is typical of Latin American countries, while the second is typical of Europe and the United States (see Rodrik (2017) for a more detailed description). In general, besides the specifics, populism is characterized by presenting a platform of apparently simple solutions to complicated problems, with the supposed objective of benefiting sectors of the population that, for various reasons, have been affected by the factors that give rise to such problems (see for instance Derks (2006)). In this context, the policies that can be implemented may have a short-term social benefit for the most disadvantaged sectors, but in the medium and long term, they could aggravate existing problems, since implementing them involves transferring resources intended for infrastructure, education, or research work, as well as implementing steep taxes on the productive sectors of society to finance immediate social policies (see Dornbusch (1991)), thus cutting off all possibilities of growth and social mobility.

Currently, it is observed in different countries around the world that politicians who aspire to be elected or re-elected to their positions usually support short-term solution policies, putting their ambitions for power before the general interest (Acemoglu, Egorov and Sonin (2013)). They
are opposed by those who maintain that the way to overcome the most pressing problems facing society is through investment in infrastructure, the development of competitiveness, research and the search for clean and economic energy alternatives that allow efficient growth without neglecting the environment. However, beyond the possible effectiveness of each of these proposals by the different political actors, the truth is that the urgent problems of the popular sectors require immediate solutions. Rodrik (2017) and Guiso, Herrera, Morelli and Sonno (2017) show that it is the need to confront these real problems that makes populist policies make sense and are supported by broad sectors of society, regardless of their level of development. It is necessary to guide the changes, looking for intelligent compensation schemes that avoid the possibility of the populist cycle, otherwise, even when citizens have opted for non-populist policies, overwhelming problems in one sector of society can lead them to support populism and then opt again for non-populism if the results are not as expected.

In this work, following the line developed by Bianchi, Favaretto and Carrera (2020), we will analyze the evolution of citizen support towards populist and non-populist politics over time. For this purpose, as in the aforementioned study, we will consider a evolutionary model from the field of games. However, we will distance ourselves from what was done by Bianchi, Favaretto and Carrera (2020), by considering that the main denotator of citizens' support for the different political currents is their ideology and the level of satisfaction that they obtain when they face politicians with the same or different ideology (populist or non-populist) rather than their status (poor or rich, employed or subsidized by the government). Furthermore, in our work we focus on the characteristics, on the parameters of the model that we propose, that can give rise to the appearance of alternate cycles of government between populist periods and non-populist periods (which we will call cycles of populism, see section 5.2).

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 introduces a two-population normal-form game that represents the pure strategies of the different populations immersed in the conflict of the democratic alternation of power that confronts citizens and the political class. Section 4 analyzes the Nash equilibria available for the game depending on the values of the parameters expressed in the return matrix of the game. Section 5 Introduces the replicator dynamics to analyze the evolution of the behavior of the citizens and politicians. In this section, we consider the possibility of cycles of alternance between populist and non-populist ruler elites. The possibility of populism and
non-populism cycles is the main contribution of our model, which makes it superior to previous models in which sfi equilibria ended up being asymptotically stable and therefore ending in a homogeneous society, which does not seem to be a reality even in the long term. Section 6 gives some conclusions and refers to future study.

1. The model

We will consider a social conflict between two populations, that of the political class, divided into populists and non-populists and that of the citizens (voters) also divided into two subpopulations, which we will call respectively, to simplify the notation, also as populists and non-populists. Citizens, through universal suffrage, decide the policy to follow once each government’s period ends.

1.1 The players

Beyond nuances, which could be important to consider in other areas, populist politicians are defined as those who propose immediate solutions to real problems but without considering the repercussions that these may have in the long or medium term (“populist leaders are not fully aware of institutional, law, administrative limitations and their rhetoric is grounded on changing overnight policy outcomes”, Bischi, Favaretto and Carrera (2020)). Hence, under this definition, by $p_p$ we denote the subpopulation politicians following a populist behavior, while by $p_{np}$ we will denote the subpopulation politicians that do not. In the context of our work, we assume that the non-populist population of politicians is made up of those politicians who see productive investment as the basis for solving the problems that the country faces for its growth and even as a solution to the most urgent problems affecting some social sectors.

On the other hand, we consider the population of citizens as the second participant in our model, and we assume that they can take 2 different positions: to support populist political sectors or to be on the side of the non-populist political sectors. By $c_p$ we denote the subclass of citizens following the first position, while by $c_{np}$ we represent the subpopulation of citizens who follow the second.

This division is undoubtedly a simplification of reality, but we believe that it largely represents the current political division in many countries, both developed and developing. The literature recognizes, in general, that the populist position is supported by stigmatized groups who face difficulties
in finding a positive social identity (Spruyt, Keppens and Van Droogenbroeck (2016). It also notes that people with greater discontent towards the lack of effectiveness of their political representatives, as well as those most affected by globalization (for example those people that for different reasons cannot adapt quickly to technological changes) at a given moment in time tend to support populist parties (see for example Coffé and Michels (2014), Arzheimer (2009), Spruyt, Keppens and Van Droogenbroeck (2016), Steiner, Mader and Schoen (2024)). Taking advantage of the urgency of these sectors, populist politicians postulate easy, momentary solutions in their platforms without considering the future repercussions and the real causes of the problem. The insecurity caused by free enterprise in the most vulnerable sectors, or immigration that seems to threaten the jobs of nationals or free trade are factors that encourage the offer of quick solutions and populist proposals, many of which appear loaded with a vengeful spirit against those who support the supposed causes of discontent.

On the other hand, non-populist sectors often forget the need to compensate the sectors most affected by technological change and the opening of markets. Many firms that remained in the market at the expense of subsidies or protectionism, must close their doors or change their way of production, adapting to new technologies, which means that those people who for different reasons cannot adapt quickly to these changes are affected sharply losing their jobs and their sources of income. Likewise, sectors of agriculture that must adapt their techniques to adapt to the needs of an economy that takes the environment into account are immediately affected (see for instance OECD (2023)). It is then necessary to consider compensatory policies that allow these sectors to adapt to change, and which are often absent from non-populist political platforms. See for instance Bajo-Rubio and Ya Ho-Don (2019).

1.2 The conflict scenarios and assumptions
We will assume a fixed number of politicians defined by #P, not the number of politicians in one population or another, which we understand can change over time #P = Pp(t) + Pnp(t), for all t ≥ 0. Similarly, for citizens, the total will remain fixed over time, but we assume that the number of individuals in one sector or another can change over time this can be represented by equality #C = Cp(t) + Cnp(t) for all t ≥ 0.

We will consider that the activity of the political class is driven by the desire to maintain or gain power, which they will obtain if they win the majority of the citizens’ votes. In this sense, the appearance of populist sectors is influenced by the growth of discontent among citizens due to
problems of poor distribution of wealth, corruption, and lack of employment or opportunities, for example. These problems, in the best of cases, do not have quick solutions and often survive a generalization even under the best efforts of governments and citizens. The promise of easy solutions through tax burdens, direct transfers, or creating new public jobs seems like a solution for some parts of the population in times of economic recession. Options that, in most cases, do not solve the problems in the medium or long term and end up becoming negative facts for their drivers.

Next, we will measure the level of satisfaction that the ruling elites obtain from their political actions by the number of votes that their actions determine, while the level of satisfaction of the citizens before the government of one sector or another will be measured by the individual welfare that is deduced from the current political actions or those promised by these sectors:

1. Citizens with preferences for populist parties when facing:
   - Populist governments receive satisfaction for ideological affinities with populist proclamations, or in some cases direct aid that includes bonuses or money. These citizens feel satisfied, both intellectually and materially, with this type of government, a fact that can be measured in terms of utility. This satisfaction will be represented by $u_{cp_p} = id_{pp} + c_{pp}$ composed of two ads, ideological affinity $id_{pp}$ and $c_{pp}$ that measures the degree of satisfaction with monetary transfers and government tax policies.
   - Non-populist governments will feel affected because, even in the case of receiving direct transfers, they will be in minor quantities and generally in the short term. Those in favor can find long-term satisfactions due to the construction of infrastructure and the possibilities of labor. The utility received by these citizens in case of facing a non-populist government will be of the type $u_{cp_{np}} = c_{pnp} + \alpha^k in, \ k \geq 0$. It is important to keep in mind that non-populist governments of different types offer some compensations to citizens, in terms of scholarships for personal study or those struggling with debt, this fact is measured by $c_{pnp}$ and it strongly related with government tax policy, and $\alpha \in [0, 1)$ is a discount factor associated with the benefits of long or medium term that infrastructure construction can contribute to.
the citizen, which multiplied by the amount of investment in that the non-populist government makes in this area, represents the satisfaction that the citizens with preferences by governments non-populists obtains for said investment and $k > 0$ corresponds to the time expected for the results of the investment in infrastructure.

2. Citizens with preferences for a non-populist government when facing
   A populist government will feel affected by the ideological point of view as well as in their beliefs about future intellectual and financial development. We understand that these citizens are generally related to intellectual development, free initiative and without the need for external support not based on individual achievements to achieve success, which makes them feel excluded from paternalistic policies typical of populist platforms. The utility that these citizens will receive from populist governments we represent as $u_{c_{npp}} = i_{d_{npp}} - e_{npp} + c_{npp}$, where $i_{d_{npp}} < 0$ represents the discomfort that these citizens feel before a populist government and $e_{npp} > 0$ is an index that measures the average exclusion feeling. The work carried out by the government, measured through the infrastructure carried out (communication routes, lighting modernization, road improvements, etc.), allocation of resources for low-income people or support through scholarships for students, support for mothers of family, etc., benefits the entire population regardless of the ideologies of the latter, so in our model, the vision of the non-populist population towards this kind of work by a populist government is represented by $c_{npp}$.

- Non-populist governments. During a non-populist governmental period, non-populists will feel satisfied with the creation of infrastructure that allows a development of commercial and industrial activities, and for the ideological affinity for the factors that drive the growth of the economy. We will represent this level of satisfaction for $u_{c_{npp}} = i_{d_{npp}} + \alpha^h in + c_{npp}$. Where $i_{d_{npp}}$ represents the degree of ideological affinity, $\alpha \in (0, 1]$ a discount on the discount factor in the creation of infrastructure and the exponent $h < k$. It is noteworthy that the infrastructure investment carried out in the government period does not necessarily imply benefits for society in the government period, but in the future, even in this situation, the work developed by the government is valued by the different members of society, this is represented by $c_{npp}$. 

Note that we do not make assumption about the sign of $c_{npp}$ and $c_{nnpn}$, nor of their relationship between it, but we claim that these are negative whenever the assessment of the government’s actions (populist or non-populist) is not well perceived by different types of citizens and positive when it is well received.

3. Populist governments facing:
   - Populist citizens: In general, populist platforms will be favored by the vote of these citizens. Fidelity will be increased by the level of promised aid. We will measure the utility of the government by $u_{gpp} = \beta_1 c_{pp} + \beta_2 i_d_{pp}$. The utility will be measured by the possibility of vote, which depends proportionally on the current promises or subsidies that are offered on the populist platform and the ideological affinity that these popular sectors feel with the populist elite.

   - Non-populist citizens: While these sectors are not prone to populist policies, some intellectuals concerned about the problems that affect the most vulnerable sectors of society and expect immediate solutions for them without taking into account the possible negative consequences that in the long term, economically, a populist policy can have, or get worse even in the case of consolidating (see Funke, Schularick and Trebesch (2021)). We will measure the utility of the populist government for the resonance that its promises achieve in this sector of intellectuality. To achieve such a result, they must present their proposal as a valid alternative to better the situation of those who are in the worst conditions.

4. Non-populist government facing:
   - Populist citizens must convince them that there are no easy solutions and that the deep causes of difficulties are not necessarily apparent and that the easy solutions offered are generally the only ones indicated by populist sectors. Notwithstanding the degree of difficulty that these sectors of the population find in their day to day can lead to unfavorable responses from these sectors directed towards the non-populist elites. There is little probability that an individual from the group in progress feels favored by the populist platform. The utility that this political sector will obtain from its relationship with this sector of the population will correspond to the probability of
convincing them about its proposals, then equal to $u_{g_{npp}} = \gamma CC$ where $CC$ measures the ability to convince.

- Non-populist citizens: The probability that the citizens of the group vote for this sector of the political spectrum will depend on their proposals for the growth and development of the country and its ideological proposal, especially the elements related to government efficiency. $u_{g_{npp}} = \gamma pr$, with $\gamma$ next to 1, by $p$ we represent the clarity of the proposal presented by the candidates for the government or to be reelected.

2. The game

It is a two-population normal form game that is repeated. The utility associated with each possible strategy of the political class will be defined by the expected quantity of votes it presents. While the utility for citizens is measured by the expected value of the degree of well-being that the proposed political platform supposedly offers.

For this we will consider $u_{g_{pp}}, u_{g_{pnp}}$ as the value of the strategy $p_p$, that is, the ability to win the vote of an individual of the classes $p$ or $np$ when a populist platform. Similarly, $u_{g_{npp}}, u_{g_{npnp}}$ represent the ability to win the vote of one individual from each class when presenting a non-populist platform, that is when the elite ruler follows strategy $P_{np}$.

Citizens of the populist sector ($C_p$) expect to reach a level of welfare $u_{c_{pp}}, u_{c_{pnp}}$ depending on whether a populist or a non-populist government win. Similarly, $u_{c_{npp}}$ and $u_{c_{npnp}}$ represent the expected welfare values for non-populist citizens, in the case that a populist or non-populist government wins respectively.

The return matrix of the game is given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$C_p$</th>
<th>$C_{np}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_p$</td>
<td>$u_{g_{pp}}, u_{c_{pp}}$</td>
<td>$u_{g_{pnp}}, u_{c_{npp}}$</td>
</tr>
<tr>
<td>$P_{np}$</td>
<td>$u_{g_{npp}}, u_{c_{pnp}}$</td>
<td>$u_{g_{npnp}}, u_{c_{npnp}}$</td>
</tr>
</tbody>
</table>

where
\[\begin{align*}
\text{u}_gpp &= \beta_1 c_{pp} + \beta_2 id_{pp}, \text{u}_g_{npp} = \gamma \text{Re}, \text{u}_g_{npn} = \gamma CC, \text{u}_g_{nppn} = \gamma Pr, \\
\text{u}_c_{pp} &= id_{pp} + c_{pp}, \text{u}_c_{npp} = id_{npp} - e_{npp} + c_{npp}, \text{u}_c_{ppn} = c_{np} + \alpha^k in, \\
\text{u}_c_{nppn} &= id_{nppn} + \alpha^h in + c_{nppn}.
\end{align*}\]

According to Von Neumann's expected utility theorem, a rational player will choose the strategy that offers the highest expected value.

\[E(P_p) - E(P_{np}) = x_{c_p}(\beta_1 c_{pp} + \beta_2 id_{pp} - \gamma CC + \gamma Pr - \gamma \text{Re}) + \gamma \text{Re} - \gamma Pr \]  

\[E(C_p) - E(C_{np}) = x_{c_p}(id_{pp} + c_{pp} + e_{npp} - id_{npp} - c_{npp}) + id_{nppn} + \alpha^h in + c_{nppn} - \alpha^k in - c_{nppn} \]

Where by
\[x_{c_p}(t) = \frac{c_p(t)}{\#C}, x_{c_{npp}}(t) = \frac{c_{npp}(t)}{\#C}, x_{P_p}(t) = \frac{P_p(t)}{\#P}, x_{P_{np}}(t) = \frac{P_{np}(t)}{\#P}\]

we denote the respective percentage of individuals in each subpopulation at each time. Note that the expected value depends on time however, to simplify notation we omitted writing the variable \(t\) in the previous expressions. In addition, in that which follows, in order to save notation, we set:

\[A_{pp} = \beta_1 c_{pp} + \beta_2 id_{pp} - \gamma CC + \gamma Pr - \gamma \text{Re}, \]
\[B_{pp} = \gamma (\text{Re} - Pr), \]
\[A_{cp} = id_{pp} + c_{pp} + e_{npp} - id_{npp} - c_{npp} + id_{nppn} + \alpha^h in + c_{nppn} - \alpha^k in - c_{np}, \]
\[B_{cp} = c_{np} + \alpha^k in - id_{nppn} - \alpha^h in - c_{nppn}. \]

According to our initial assumptions, although the number of individuals in each population (politicians and citizens) is considered constant in the model, this is not the case with the percentages of individuals in each subpopulation since these percentages can change over time. Furthermore, the supposed rationality of the participants implies a growing percentage of individuals in the subpopulation whose strategy has a higher expected value. Note that this does not necessarily imply that the strategy that once was successful will continue to be so. For example, in the event that individuals waiting for direct transfers increase in number, the budget of the State is finite, which means that more and more
resources are required to maintain the level of aid, or that the amount of each aid is reduced, all of which can result in a change in the attitude of the voters.

For each of the possible strategies there are restrictions. One of the most important constraints is the government’s budget constraint, this means that the total possible aid that a populist government can offer cannot exceed revenue (we do not consider the possibility of the government borrowing abroad). Therefore, even a populist government must find some source of resources or reduce the aid provided. Many times, the resources allocated to subsidies come from cuts in basic services, education and health, as well as the cutting resources destined to infrastructure, or the collection of taxes in particular to the productive sectors of society. The need for infrastructure can become a critical factor for the growth of the country, the neglect of communications and transport and in general, the lack of basic services, can make even populist governments feel obliged to allocate resources for the creation of said infrastructure. Investment that will often be made at the expense of previously granted aid, inflation or indebtedness, are factors that can lead the country to financial crises and end up harming society as a whole. This situation will once again put citizens into consideration for the need of governments that address the problems of growth and economic balances. On the other hand, a heavy tax burden can harm the productive sectors, causing shortages even of basic necessities, all of which will lead citizens, including those who originally supported the government, to reconsider their vote for the next period. Argentina during different Peronist governments is a clear example of this situation (see for example OECD (2019)).

Similarly, non-populist governments will face budget constraints. Inefficient investment in public services and infrastructure in general, can generate financial difficulties, which in turn cause difficulties in the most vulnerable sectors of the population, unemployment and lower quality of life, as well as discontent in sectors of the population linked to industry and commerce that require efficient investments in services and infrastructure for the normal development of their activity. On the other hand, economic growth as a result of investment in: science and technology, the opening of markets, the incorporation of new technologies, competition with international firms and caring for the environment can generate unemployment and unexpected changes with negative repercussions for some sectors of the population (see Accinelli and Muñiz (2021)). These changes in the absence of an adequate public policy can generate discontent and unrest in the population even when
government policy is aimed at improving the performance of the economy and economic growth. This situation may cause changes in the electoral decisions of a large part of the citizens who until now identified with the government, opening the door to the presentation of populist platforms.

3. The Nash Equilibrium of the political game

While a Nash equilibrium in pure strategies assumes that the entire political class behaves in a single way, and the same occurs for the population of citizens, a mixed strategy corresponds to a percentage distribution of the individuals of each population among its possible subpopulations (this equilibrium is strictly mixed if both distributions are not degenerated). The latter corresponds to a certain situation of uncertainty, where it is not clear to the political class what the preferences of citizens are, which means that not all politicians act in the same way. As we will see, for this type of equilibrium the model presents two alternatives. The first one corresponds to a stable equilibrium, in the sense that the distribution over time between the subclasses can be modified, if we start close to the equilibrium distribution, society will change a lot, cycles will result in which political elites of one ideology and another alternate in power, with the support of the majority of citizens. Or the second possibility corresponds to the case where the strictly mixed Nash equilibrium is unstable, in the sense that it will be difficult for society to reach it, but even if the respective distributions are at some point close to the equilibrium distributions, they will move away from them forever. We will consider these possible scenarios in the next section; however, the evolution can only be considered once we introduce a dynamic for the game, which will be done later in section 5.

3.1 Nash equilibria in possible political-ideological scenarios

In this section we discuss some possible scenarios to which the different political options can give rise considering the declared objectives and the perception of the citizens on the results of the same. Although it is possible in principle to characterize economic policies in two large categories as we have done up to now, the results obtained by governments that fall into one or the other are not necessarily the same or affect citizens in the same way. This means that citizens do not appear as a homogeneous whole, and even under the same government, the citizens’ perception of the results of government policy may be different, which in principle means that we can place them in two large groups, the sympathetic to the government, those who see its politics will improve their standard of
living, and the non-sympathetic, those who feel they will be harmed by
the same policy.

Since the ruling elite intends in principle to remain in power, it will
require the majority vote in democratic countries, therefore it will be
attentive to citizen opinion, and it may turn out that it does not always act
as a populist party even if it is, or even when it does not make decisions
that are characterized as populist. This causes the percentages of citizens
in one or the other group to change over time, which in turn leads to
possible variance in government actions. The result is that the political
sector with which the majority of citizens feel identified ends up
triumphing. In other words, if populist policies are perceived by the
majority as causing bad results, the choice of the citizens will fall on the
non-populist elite, if this policy is perceived as successful by the majority,
the populists will be re-elected. The same will happen for non-populist
governments.

We will show different scenarios, which correspond to different game
structures that characterize our model, structures that give rise to
different return matrices and, consequently, to different sets of Nash
equilibria. In other words, each of these scenarios will ultimately be
classified by a set of Nash equilibria.

3.2 Scenario with several Nash equilibria

Under these conditions it is not possible, at least in principle, to decide
which of the possible Nash equilibria will end up or be enhanced in
society, at least not, as long as we do not incorporate a dynamic into the
model that allows us to decide on the stability of these equilibria. In short,
only stable equilibria are observable.

**Theorem 1.** If \( u_{cnpp} > u_{cnp} \) and \( u_{cp} > u_{cnpp} \) together with \( u_{g_{npp}} > u_{g_{pp}} \) and \( u_{g_{pp}} > u_{g_{npp}} \) holds, then the game has three Nash equilibria:

i) *(Pure Nash equilibrium)* populism - populism.

ii) *(Pure Nash equilibrium)* non-populism - non-populism.

iii) *(Strictly mixed Nash equilibrium)*

\[
\begin{align*}
x_{cp} &= \frac{-B_{pp}}{A_{pp}}, x_{cp} &= \frac{-B_{cp}}{A_{cp}}
\end{align*}
\]

Before presenting the proof of the theorem we remark that first
equilibrium can be identified with the probability distribution \((x_{cp}, 1 – \)
which for short we write \((x_p, 1 - x_{pp}) = (1, 0), (0, 0)\), which means that no player is taking populist behavior, hence they act in a non populist way.

**Proof.** See figure 1, since the dynamics of the edges of the unit cube correspond with the best response of each player against the alternative that the other is playing, then the vertices with two arrows pointing in their direction correspond to Nash equilibria in pure strategies, so \((0,0)\) and \((1,1)\) are the only pure Nash equilibria of the game. Certainly the expression (4) satisfies \(E(P_p) - E(P_{np}) = 0 = E(C_p) - E(C_{np})\), so to guarantee that this is a Nash equilibrium in mixed strategies, we must prove that \(0 < x_{cp}, x_{pp} < 1\). To see this note that the assumptions of the theorem implies that \(u_{g_{npnp}} - u_{g_{pp}}\) and \(u_{g_{pp}} - u_{g_{npp}}\) are positive, hence the result follows from the fact that \(-B_{pp} = u_{g_{nnp}} - u_{g_{np}}\) and \(A_{pp} = u_{g_{pp}} - u_{g_{np}} - B_{pp} > -B_{pp}\), then \(0 < x_{cp} < 1\). A completely analogous argument shows that the assumptions of the theorem together with the definition of \(B_{cp}\) and \(A_{cp}\) implies that \(0 < x_{pp} < 1\).

The theorem describes the general conditions for which populist citizens have greater satisfaction when political leaders have the same ideologies as them, and vice versa, that is, populist leaders have a higher utility when backed by citizens with their same interest. In the same way it reflects the conditions under non-populist leaders if non-populist citizens have no interest in changing their political ideologies unilaterally.

### 3.3 Scenarios with only one Nash equilibrium

**Theorem 2.** Assume that the inequalities \(u_{c_{npp}} > u_{c_{npn}}, u_{g_{pp}} > u_{g_{npp}}, u_{c_{npp}} > u_{c_{pp}}\) and \(u_{g_{nnp}} > u_{g_{np}}\) hold, then the only Nash equilibrium of the game is strictly mixed and is given by the expression

\[
x_{cp} = -\frac{B_{pp}}{A_{pp}}, x_{pp} = -\frac{B_{cp}}{A_{cp}}
\]

The same result holds whenever all the inequalities in the hypothesis of the theorem are in the opposite sense.
**Proof.** Cada vértice del enunciado de la figura 2 tiene un flecha apuntando hacia él y otra saliendo de él, ya que las dinámicas de los bordes del cuadrado unitario corresponden a las mejores respuestas de cada jugador al alterno que podría jugarse por el otro, podemos concluir que la mejor respuesta para cada estrategia jugada por el otro jugador es contrario al alterno elegido por el primero, entonces el juego no tiene un equilibrio en estrategias puros, luego por el teorema de Nash para juegos de estrategias finitas, podemos concluir que debe existir un equilibrio en estrategias mixtas. Dado que

\[ x_{cp} = -\frac{b_{pp}}{A_{pp}}, \quad x_{pp} = -\frac{b_{cp}}{A_{cp}} \]

tenemos que

\[ E(P_p) - E(P_{np}) = 0 = E(C_p) - E(C_{np}), \]

entonces la conclusión del teorema se sigue de la definición \(A_{pp}, B_{pp}, A_{cp}, B_{cp}\) y la hipótesis del teorema que garantiza que \(x_{cp}, x_{pp} \in (0,1)\).

\(uc_{pnp} > uc_{nnpp}\) es equivalente a \(\alpha^k in + c_{pp} > id_{npn} - \alpha^h in + c_{nnpp}\) este caso puede ser en un caso donde incluso con ciudadanos no populistas con preferencia a populismo tienen una utilidad mayor que los ciudadanos no populistas con un gobierno no populista. Esto puede ser porque las acciones o políticas seguidas por gobernantes no populistas no han sido lo suficientemente exitosas en cumplir con las expectativas que los ciudadanos no populistas tenían. Es posible que la conservación de algunas políticas de subsidio haya mantenido la utilidad de ciudadanos populistas a un nivel más alto, incluso cuando prefieren a un gobierno populista. En este caso, incluso cuando la ideología de los ciudadanos no populistas es la misma que la del gobierno, es decir, incluso cuando \(id_{npn} - \alpha^h in > \alpha^k in\), la valoración de los ciudadanos no populistas sobre las acciones del gobierno no populista es contraria a sus ideales políticos, mientras que la valoración de los ciudadanos populistas a la performance del gobierno no populista es positiva, en el sentido de que estos ciudadanos consideran que el trabajo del gobierno no populista ha sido adecuado. En este caso \(c_{pnp} > c_{nnpp}\) desigualdad que puede ser interpretada como el gobierno populista está llevando a cabo una política fiscal más cercana a los sectores no populistas que a los populistas. Las razones para esto pueden ser varias, entre ellas, la necesidad de fomentar la producción del país o sus exportaciones, sin lo cual no es posible sostener largos plazos populistas. La utilidad de los ciudadanos no populistas es \(uc_{nnpp} > uc_{pp}\) es equivalente a \(id_{npn} - e_{npp} + c_{npp} > id_{pp} + c_{pp}\), lo que refleja el trabajo del gobierno populista para resolver el problema de población, siguiendo su ideología y llevando a cabo actos que valoren a esos ciudadanos con ideologías contrarias a las suyas, es decir, \(c_{npp} > c_{pp}\). ¿Qué puede reflejar que los programas creados para infraestructura, creación de servicios públicos, asignación de becas, etc., han sido más beneficiosos para ciudadanos no populistas que para ellos.
than for the populists. This may be due to the inability of the populist government to deliver on all its promises in the short term, which is reflected in the disappointment of like-minded citizens of the government. While the perception of the rest of the citizens is that of a government that works to solve the underlying and long-term problems. This does not mean that the government has changed its populist thinking, rather it reflects the reality it faces, where budgets may be insufficient to solve problems overnight and strategies to do so will take longer than the populists thought.

\[ u_{g_{npn}} > u_{g_{ppn}} \text{ and } u_{g_{pp}} > u_{g_{npp}} \], these inequalities capture the actions of the government, who, seeing the preference of the citizens, measured for example through surveys, takes the position that ensures it wins over the electorate, making populist campaigns when it warrants it (when facing populist citizens) and non-populist campaigns when needed (facing non-populist citizens).

4. Replicator dynamics

Replicator dynamics reflect how the most successful behavior is replicated. Its origin is in biology in which the behavior that best adapts to the environment is the one that eventually prevails. Individuals that act according to this behavior tend to reproduce more easily. In society, the most successful behavior is replicated or the behavior that individuals understand as such at any given time. Unlike behavior in nature, in society this is not genetically determined, but rather obeys a rational choice made based on the information that each individual has at the moment in which they must choose one behavior or another. Learning, imitation or maximization of an expected value considering the probabilities of success at each moment determine the dynamics, see for example “Corruption driven by imitative behavior” Accinelli and Carrera (2012).

Each voter will vote for that sector that, given the information about him at the time, offers him the greatest utility, regardless of his previous affiliation.

Our model intends to analyze the alternation in power of the elites that are defined by one or another policy. Until now the models were static, however they tried to show the possibility of alternate cycles. The replicator dynamics models the learning of rational individuals who consider the expected value of their possible strategic choices. The
The analysis of the stability of their equilibria makes it possible to define, in the event of the existence of multiple Nash equilibria, which one ends up prevailing, since in reality only stable equilibria are foreseeable. It also makes it possible to analyze the appearance of cyclical trajectories centered on one of its equilibria. It is precisely this analysis that this section will deal with.

In our model the replicator dynamics is described by the following system of differential equations:

\[
\begin{align*}
\frac{dx_c}{dt} &= x_c \left(1 - x_c\right) \left(E(C) - E(C_{np})\right) \\
\frac{dx_p}{dt} &= x_p \left(1 - x_p\right) \left(E(P) - E(P_{np})\right)
\end{align*}
\]  

Here \(E(C)\) and \(E(C_{np})\) represent respectively the expected value of playing \(C\) and \(C_{np}\) by citizens, while \(E(P)\) and \(E(P_{np})\) denotes the expected value of the politics playing \(P\) and \(P_{np}\). Then using (2) and (1) system (6) becomes in

\[
\begin{align*}
\frac{dx_c}{dt} &= x_c \left(1 - x_c\right) \left(x_{cp} A_c + B_{cp}\right) \\
\frac{dx_p}{dt} &= x_p \left(1 - x_p\right) \left(x_{cp} A_{pp} + B_{pp}\right)
\end{align*}
\]  

Note that the Nash equilibrium corresponds with stationary states of the system, but the converse is not necessarily true. In section 4 in theorem 2 and theorem 1 we present some general conditions and its explanation about the political realities of the society that implies the existence of one or multiple Nash equilibria.

For the multiple Nash equilibria case using classical game theory, it is not possible, at least in principle, to indicate which of the Nash equilibria of a game ends up prevailing. The study of the stability of the equilibria of replicating dynamics, considered in the modern field of evolutionary games, will allow us to define which of them will end up prevailing. In short, both in society and in nature, only stable equilibria are perceptible. We will make the study of the stability of the multiple equilibria case in subsection 4.1.

It is important to take into account the equations of the dynamical system corresponding to those that guarantee the existence of the solution. Its uniqueness will be determined once we know the initial conditions. that is, a distribution over the possible subpopulations for each population at a given moment. Once these distributions are known, they find the only possible trajectory that the evolution of the system that corresponds to such initial conditions will follow. We will call these initial conditions the
initial state of the system, while the state of the systems at each moment corresponds to the pair of distributions existing at each moment, which certainly mark the political evolution of society.

4.1 The stability analysis of the multiple equilibria scenario

Using the Hartman-Grobman theorem in this section we analyze the stability of the strictly mixed Nash equilibrium (4). For this purpose, we estimate the eigenvalues and its signs of the linearization of the system (7). From a direct computation the Jacobian matrix of system (7) is given by

\[ J(x, y) = \begin{bmatrix}
(1 - 2x) (y + B) & x (1 - x) \\
(1 - 2y) (x + B) & (1 - 2x) (y + B)
\end{bmatrix} \] 

Evaluating the Jacobian matrix (8) of system (6) at the strictly Nash equilibria \((x_*, y_*)\) given by (5) or (4) we obtain

\[ J(x_*, y_*) = \begin{bmatrix}
0 & x^* (1 - x^*) \\
x^* (1 - y^*) & 0
\end{bmatrix} \] 

Even when the same Jacobian matrix is obtained for the Nash equilibria described by (5) and (4) the dynamical behavior of both is completely opposite. In the next theorem we present the stability analysis of the strictly mixed Nash equilibrium for the scenario of multiple equilibria. We leave the analysis of the scenario of unique strictly Mixed Nash equilibrium to the end of this section.

**Theorem 3.** Under assumption of theorem 1 the strictly mixed Nash equilibrium

\[ (x_*, y_*) \]

is an hyperbolic equilibrium point of the system (7).

**Proof.** Finding the eigenvalues of (9), we obtain

\[ \lambda = \pm \sqrt{x^* (1 - x^*) x^* (1 - x^*) A_{pp} A_{cp}} \] 

\[ (10) \]
since $x_{cp}^*, x_{pp}^* \in (0,1)$ and the hypothesis of the theorem implies that $A_{pp} > 0$ and $A_{cp} > 0$ (see proof of theorem 1) the theorem follows.

**Observation 1.** Note that under the conditions of last theorem $J\left(x_{cp}^*, x_{pp}^*\right)$ has two real eigenvalues of opposite sing and same module. Then the strictly mixed Nash equilibrium $(x_{cp}^*, x_{pp}^*)$ is a saddle point for the replicator dynamics with a 1-dimensional unstable manifold and a 1-dimensional stable manifold passing through it. Hence the probability of convergence to the equilibrium in the interior of the unit square is equal to zero.

One can easily conclude that the Nash equilibria $(0, 0), (1, 1)$ are attractors of the dynamical system, while the equilibria states $(1, 0)$ and $(0, 1)$ are saddle points. Before to continue we recall that these equilibria correspond respectively with the strategic profiles $(C_{np}, P_{np}), (C_p, P_p), (C_p, P_{np}), (C_{np}, P_p)$.

Once we set the initial condition of the system, the way in which populism evolves is determined, therefore, from the previous observation, the behavior of politicians and citizens that eventually converges towards one of the system’s attractors. Dynamic, depending on the location of the initial distribution of citizens and politicians. That is, the percentage behavior of citizens and politicians will converge towards one or another of the balances. That is, towards a balance with a high level of populism (1.1) or towards a very low level of populist acts (0.0).

As we point out above, the Jacobian matrix at the strictly mixed Nash equilibria described in theorem 1 and theorem 2 is the same for both cases. Unfortunately, under the condition of theorem 2 the sign of $A_{pp}$ and $A_{cp}$ is opposite being positive for the first constant and negative for the second. Hence in this case we can not apply the Hartman-Grobman theorem, since the eigenvalues of the Jacobian matrix are imaginary pure. Even though the equations defining the replicator dynamics allows us to study the behavior of the dynamica for this case, allowing us to conclude the existence of a cycle as is described in the next section.
In democratic societies, homogeneity of thought is not a widely spread characteristic. This does not mean that although, according to what has been analyzed so far, there are homogeneous equilibria, rather this should be considered as a trend, which can be reversed if any of the model parameters are modified, and with it or them, the basin of attraction of one is modified. However, it is important to note that even without changes in the parameters the possibility of cycles of alternating governments seems to be a real possibility. Our model, unlike previous models, clearly shows the possibility of such cycles emerging.

4.2 Populist, non-populist Cycles

Most of the specialized literature, see for example Dornbusch (1991), indicates that populist policies are based on expansive monetary policies that cause inflation and economic imbalances as well as debt to face non-productive transfers, lack of investment in the sectors of production and public services, loss of competitiveness of the economy and disinterest in improving on the part of broad sectors of the population. These, among other calamities, would give rise to an economic crisis, which in the end causes populist policies to end up harming, even those who they initially claimed to represent. This makes the majority of citizens look for an alternative, choosing populist policies in the medium term.

Although non-populist policies can give rise to growth and development, attention is needed because of the fact that many times these factors create inequalities which can give rise to feelings contrary to these
policies in large sectors of the population. This can lead to the search for alternatives, which would be populist politics.

A populism cycle is characterized by the following four equations, which holds under the conditions of theorem 2.

- Citizens become populist when the political parties have stopped seeing it:
  \[ \dot{x}_{cp} = x_{cp} \left( 1 - x_{cp} \right) B_{cp} > 0. \]  
  \( (11) \)

  This can happen at election time when the results of the traditional parties that are not populist have ceased to convince the people, and when politicians present proposals for short-term solutions, although these may be unrealistic or difficult to achieve, these are the solutions that voters expect.

- The behavior of politicians becomes more and more populist when they observe that citizens prefer electoral populism. This scenario is summarized in the equation
  \[ \dot{x}_{pp} = x_{pp} \left( 1 - x_{pp} \right) (A_{pp} + B_{pp}) > 0. \]  
  \( (12) \)

  A more detailed explanation is as follows: to ensure the preferences of the electorate, politicians run campaigns based on what they understand to be the aspirations of the majority of citizens. To get to know them, they invest in political advisors who advise on the political platform and electoral discourse. So, it is not surprising that in a scenario where the society faces high-impact problems such as inequality, immigration or the lack of decent jobs, the electorate prefers platforms that promise short-term solutions, instead of those that propose long term recovery plans more related to productive investment rather than social subsidies. This scenario gives rise to populist platforms with popular slogans and giving immediate solutions to the difficult problems facing society. The populist alternative then appears as a real possibility.

- Faced with a panorama where populist politicians predominate, people change their way of seeing politics by taking a less populist behavior:
  \[ \dot{x}_{cp} = x_{cp} \left( 1 - x_{cp} \right) (A_{cp} + B_{cp}) < 0. \]  
  \( (13) \)
Faced with a panorama where populist politicians predominate, citizens can change their view of politics by adopting less populist behavior. The economic crises that harm non-populist sectors and the inability of populist policies, based on the collection and use of public resources to make transfers with the alleged intention of solving the problems of the most vulnerable sectors of society, make the citizens look for alternatives other than a populist government. The lack of incentives for investment, the inadequate use of resources allocated to public services, the lack of medicine and quality jobs and, in general, the lack of incentives for production make social sectors that previously supported populist policies now feel the need of a paradigm shift. For a more detailed explanation of this situation see for example Rodrik (2017). This trend is reflected by the following equation

$$\dot{x}_{pp} = x_{pp} (1 - x_{pp}) B_{pp} < 0.$$  \hspace{1cm} (14)

Once in this scenario the cycle restarts.

**Figure 2:** The phases diagram for the dynamics of populism.

An interesting question is how to break the populist cycle? We understand that for this purpose is necessary to consider compensatory public policies that mitigate the situation of the citizens that are most affected in the area of growth. From employees and previously protected firms, public officials whose work was the result of political clientelism. For instance, in Latin American countries, public positions resulting from clientelism are an important part of the total number of jobs: for example, people who perform tasks at the state level represent 8% of the total population and 18.2% of the people occupied in Argentina (according to
estimates based on data from the National Institute of Statistics and Censuses -INDEC- corresponding to the National Census of Population, Households and Housing 2022 and the Permanent Household EPH of the fourth quarter of 2022). The policies of the new Milei government will undoubtedly affect many of these employees, many of whom gained their positions as a result of the populist policies from previous governments. Even when they have voted for the new government, the lack of an adequate compensatory policy can lead them to vote for the previous ruling elites, thus beginning a cycle of alternation in power.

Conclusiones

In this work, we have presented a model based on the theory of evolutionary games that allows us to analyze the trajectories that the behavior of citizens and politicians will follow in a democratic country, over time and in different real socio-economic scenarios in which they face two alternative political platforms.

For simplicity we have called these alternatives populist and non-populist, however we understand that, although the simplification leaves aside nuances and may be somewhat schematic, it well represents the current division between the different economic and political proposals at a time when the old left-right alternative seems to be disappearing. On the other hand, we understand that although reality is much richer in nuances, this simplification allows us to create a model to analyze the factors that determine political alternation in a democratic society in which citizen participation ultimately decides.

In theorem 1 we show and interpret the general conditions that imply the appearance of cycles of populism/non-populism. Although in this case the Hartman-Grobman theorem does not allow us to conclude anything about the stability of the equilibrium in mixed strategies, the system of differential equations describing the dynamics of the replicator is the sufficient tool to show the evolution of the behavior of citizens and politicians towards populism. In this case, the analysis allows us to conclude the appearance of cycles central to the mixed Nash equilibria of periods of low-level populism on the part of both citizens and politicians, followed by the appearance of populist behavior on the part of one of the two, achieved after this, periods of high populism of both, and so on.

On the other hand, theorem 1 allows us to find conditions that guarantee the existence of multiple Nash equilibria in the model, for this scenario the Hartman-Grobman theorem allows us to carry out the analysis of the
stability of the mixed equilibrium, allowing us to conclude that under these conditions there is the possibility that both types of populist and non-populist behaviors coexist in periods of time. However, depending on the location of the initial conditions (percentages of citizens and populist politicians), the individual behavior of both actors in politics will converge to only one of these behaviors, that is, eventually the percentage of populist individuals will be extremely low or will converge to totally populist behaviors.

Referencias


