Ensayos–Volumen XXIX, No. 1, mayo 2010, pp. 1-22

The Theory of Storage and Price Dynamics of Agricultural Commodity Futures: the Case of Corn and Wheat

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Fecha de recepción: 14 XII 2009 Fecha de aceptación: 23 II 2010

Abstract

Using a restricted version of the BEKK model it is tested an implication of the theory of storage that supply-and-demand fundamentals affect the price dynamics of agricultural commodities. The commodities under analysis are corn and wheat. An interest-storage-adjusted-spread was used as a proxy variable for supply-and-demand fundamentals to test the aforementioned implication for both commodities. It is also tested the Samuelson hypothesis that spot prices have higher volatility than futures prices. It is found that the interest-storage-adjusted-spread has had a statistically significant positive influence on the spot and futures returns for both commodities. Likewise, the results also show that spot price returns have higher volatility compared to futures price returns which is consistent with the Samuelson hypothesis. The results of the aforementioned tests are consistent with both theories and with the existing literature related to commodity futures.

Keywords: Agricultural commodities, BEKK model, multivariate GARCH, Samuelson hypothesis, theory of storage.
JEL Classification: C22, G10, Q14.

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The opinions in this paper correspond to the author only and do not necessarily reflect the point of view of Banco de México.
The author thanks two anonymous referees for very valuable comments, the Committee from the European Financial Association (EFA) Conference in Berlin for selecting this paper as a "Discussion Meeting Paper," the Department of Accounting and Finance at Lancaster University, and CONACYT (National Council of Science and Technology) Mexico, for the generous financial support given. All remaining errors are responsibility of the author.

Resumen

Se utiliza una versión restringida del modelo BEKK para poner a prueba una implicación de la teoría de almacenamiento, la cual establece qué fundamentos de oferta y demanda afectan la dinámica de precios de productos agropecuarios. Los productos analizados son el maíz y el trigo. Se utiliza un diferencial-ajustado a la tasa de interés y a costos de almacenamiento para tener la "proxy" de los fundamentos de oferta y demanda para los bienes agropecuarios previamente mencionados. También se pone a prueba la hipótesis de Samuelson, la cual argumenta que la volatilidad de los precios spot es mayor a la volatilidad de los precios de los futuros. Los resultados muestran que el diferencial-ajustado tiene una influencia positiva estadísticamente significativa sobre los rendimientos de los precios spot y de futuros para ambos productos agropecuarios. De la misma manera, los resultados también muestran que los rendimientos de los precios spot tienen mayor volatilidad si se comparan con los rendimientos de los precios de los futuros, lo que es consistente con la hipótesis de Samuelson. Los resultados de las pruebas mencionadas son consistentes con ambas teorías y la literatura existente en relación a futuros agropecuarios.

Palabras Clave: productos agropecuarios, modelo BEEK, modelo multivariado GARCH, Hipótesis de Samuelson, teoría de almacenamiento.
Clasificación JEL: C22, G10, Q14.

Introduction

Return variability in spot and futures prices have been analyzed using multivariate GARCH models in different types of studies. McCurdy and Morgan (1991) analyzed uncovered interest rate parity. Chan, K. *et al.* (1991) made a similar study using stock and futures indices. Ng and Pirrong (1994) analyzed joint dynamics of spot and futures prices returns for metals. Jacobs and Onochie (1998) work was done for the relationship between return variability and trading volume in international futures prices. In this paper the work of Ng and Pirrong (1994) is extended to describe the joint dynamics of the spot and futures prices returns for agricultural commodities, specifically corn and wheat. This is important in order to test existing futures markets theories and to compare the results obtained here to those documented in the literature. The main contribution to the literature is to add empirical evidence about the usefulness (validity) of theories for agricultural commodities futures.

A restricted version of the Engle and Kroner (1995) multivariate ARCH model, henceforth the BEKK model, is applied to test financial theories. The

BEKK model (named like this after an earlier working paper by Baba, Engle, Kraft and Kroner) with a spread effect is applied to test specifically the theory of storage and the Samuelson's effect. Contrary to metal commodities the agricultural commodities' spot and futures prices under analysis show that the series were stationary I(0) thus, the Error Correction Model (ECM) used by Kroner and Sultan (1991) and Ng and Pirrong (1994) could not be used in this case. Hence the use of the BEKK model is a reliable alternative estimation. An additional novelty includes the relatively larger sample of almost twenty-five years of daily data for both commodities under analysis. Aforementioned research papers used weekly data for a significantly smaller sample period.

The layout of the paper is as follows. Section 1 details about futures market theories and the methodology applied here to analyze them. Section 2 presents the model. An explanation about the data and its transformation is presented in Section 3. Descriptive statistics and data analysis are presented in Section 4. In Section 5 there are analysis of the results. Finally, there is a conclusion.

1. Details of the theories and research methodology

1.1. The Theory of Storage

The Theory of Storage by Kaldor (1939), states that the spread between spot and futures prices is determined by fundamental supply-and-demand conditions. Specifically, the behavior of commodity futures and spot prices are related to storage costs, inventory levels and convenience yields. Main contributions to the theory have been in three ways: 1) Analysis and evidence relating to it; 2) Empirical tests of the implications related to inventories; and 3) Empirical tests of the implications related to the behavior of the basis.[1] Among the important contributions for the explanation of the theory, in terms of theoretical analysis and including some evidence relating to it, are the ones by Working (1948, 1949), Telser (1958), Bresnahan and Suslow (1985), Bresnahan and Spiller (1986), Williams (1986), Williams and Wright (1989, 1991), Brennan (1991), Deaton and Laroque (1992). On the one hand, these works have explained mainly theoretical implications of the theory. On the other hand, in addition to theoretical explanations of the theory of storage, previous work has been done to test the implications of the theory by empirical work. A cornerstone in the literature of empirical work is shown in the seminal research papers done by Fama and French (1987, 1988) in which the conclusion is that most of the implications tested in the theory of storage hold. Other empirical works were elaborated by Brennan

[1] Basis represents the difference between the futures price and the spot price.

(1958), Cho and McDougall (1990), Ng and Pirrong (1994) and Susmel and Thompson (1997).

It is believed that inventories of agricultural commodities are held given that there is a stream of benefits to hold them called convenience as Brennan (1958) explained. Basically there are two main reasons that explain the existence of convenience, Fama and French (1987). One is that producers and/or consumers who hold the commodity physically could find benefits of having supplies (stocks) of the commodity to meet unexpected demand. The other one is that the supplies of the commodity could be used at any time as an input in a production process.

From the theory of storage it is possible to derive six testable implications on this theory.[2] In this paper one of the implications of the theory of storage will be tested. The implication is that spot and futures price volatilities are influenced by supply and demand fundamentals. Following Ng and Pirrong (1994) in this research paper convenience yields are used as a proxy variable for supply and demand fundamentals. Thus, the analysis will consider the influences of convenience yields on spot and futures price return volatilities for each commodity under analysis.

1.2. The Samuelson Effect

The Samuelson (1965) effect which states that the volatility of the spot prices is higher than the volatility of futures prices will be tested in the following way: first, average fitted values of futures return variabilities and spot return variabilities will be compared to each other. It is expected that the fitted values of the futures return variabilities will be less than the fitted values of the spot return variabilities. Secondly, a correlation coefficient between the ratio of the futures return volatility to the spot returns volatility and the convenience yield will be estimated. According to this theory the spot-return volatility must be larger than the futures-return volatility as the market becomes more inverted *i.e.* as the convenience yield increases. Therefore, it is expected that the correlation between the ratio of the futures return volatility to the spot returns volatility and the convenience yield must be negative.

1.3. No-Arbitrage Theory and the Adjusted Spread

Following Ng and Pirrong (1994) supply and demand fundamentals could be expressed as a proxy variable of an interest-storage adjusted spread between

[2] For more details about each testable implication of the theory of storage the interested reader can refer to Ng and Pirrong (1994, 208).

spot and futures prices. This could be expressed in mathematical terms considering the no-arbitrage relation between spot and futures prices in Equation (1):

$$F_{t,T} - W_{t,T} = S_t e^{(R_{t,T} - CY_{t,T})(T-t)} \tag{1}$$

In Equation (1) $F_{t,T}$ represents the futures price and S_t represents the spot price. $R_{t,T}$ is the interest rate. The variables $W_{t,T}$ and $CY_{t,T}$ represent the marginal storage cost and convenience yield respectively. The pair of subscripts t, T defines variables at time t for futures delivered at time T. Solving for the interest-storage adjusted spread (hereafter the adjusted spread) it follows that the formula to obtain the aforementioned variable is shown in Equation (2):

$$z_t = \frac{\ln(F_t - W_{t,T}) - \ln S_t}{T-t} - R_{t,T} = -CY_{t,T} \leq 0 \tag{2}$$

where z_t represents the adjusted spread. The no arbitrage theory will predict that z_t will vary directly with inventories so it will be ensured that the convenience yield will affect the relationship between spot and futures prices. Also, the theory of storage implies that spot prices will become more volatile as inventories are low, *i.e.*, convenience yields increase. To test formally the above mentioned theories an econometric model will be applied.

2. The Model

The model to be used is the BEKK model, which estimates the conditional variances and covariances of the series under analysis using a multivariate GARCH method. The procedure to obtain the aforementioned multivariate GARCH model is explained in Equations (3) through (7).

Let y_t be a vector of returns at time t,

$$y_t = \mu + \varepsilon_t \tag{3}$$

where μ is a constant mean vector and the heteroskedastic errors ε_t are multivariate normally distributed:

$$\varepsilon_t | I_{t-1} \sim N(0, H_t) \tag{4}$$

Each of the elements of H_t depends on p lagged values of the squares and the cross products of ε_t as well as those on the q lagged values of H_t. Considering a multivariate model setting it is convenient to stack the non-redundant elements of the conditional covariance matrix into a vector, *i.e.,* those elements lie on and below the main diagonal. The operator, which performs the aforementioned stacking, is known as the *vech* operator. Defining h_t is $vech(H_t)$ and η_t represents $vech(\varepsilon_t \varepsilon_t')$ the parameterization of the variance matrix is:

$$h_t = \alpha_0 + \alpha_1 \eta_{t-1} + \ldots + \alpha_p \eta_{t-p} + \beta_1 h_{t-1} + \ldots + \beta_q h_{t-q} \quad (5)$$

Equation (5) is called the *vech* representation. Bollerslev *et al.* (1988) have proposed a diagonal matrix representation, in which each element in the variance matrix $h_{jk,t}$ depends only on past values of itself and past values of the cross product $\varepsilon_{j,t}\varepsilon_{k,t}$. In other words, the variances depend on their own past squared residuals and the covariances depend on their own past cross products of the relevant residuals. A diagonal structure of the matrices α_i and β_i is assumed in order to obtain a diagonal model in the *vech* representation shown in Equation (5).

In the representations explained above it is difficult to ensure positive definiteness in the estimation procedure of the conditional variance matrix. To ensure the condition of a positive definite conditional variance matrix in the optimization process Engle and Kroner (1995) proposed the BEKK model. This model representation can be observed in Equation (6):

$$H_t = \omega\omega' + \sum_{i=1}^{p} \alpha(\varepsilon_{t-i}\varepsilon_{t-i}')\alpha' + \sum_{i=1}^{q} \beta H_{t-i} \beta' \quad (6)$$

In Equation (6) $\omega\omega'$ is symmetric and positive definite and the second and third terms in the right-hand-side of this equation are expressed in quadratic forms. This ensures that H_t is positive definite and no constraints are necessary on the α_i and β_i parameter matrices. As a result, the Eigenvalues of the variance-covariance matrix will have positive real parts which satisfy the condition for a positive definite matrix.

For an empirical implementation related to the analysis of the influence of the adjusted spread to the spot and futures return variabilities for each agricultural commodity and following Ng and Pirrong (1994) the model can be extended as in Equation (6'):

$$H_t = \omega\omega' + \sum_{i=1}^{p} \alpha(\varepsilon_{t-i}\varepsilon'_{t-i})\alpha' + \sum_{i=1}^{q} \beta H_{t-i}\beta' + \gamma z_{t-1}^2 \tag{6'}$$

In Equation (6') ω is a 2 x 2 lower triangular matrix, α and β are 2 x 2 diagonal matrices and γ is a 2 x 2 symmetric matrix. For the bivariate case the BEKK model can be expressed in vector form in Equation (7):

$$\begin{bmatrix} H_{11,t} & H_{12,t} \\ H_{12,t} & H_{22,t} \end{bmatrix} = \begin{bmatrix} \omega_1 & 0 \\ \omega_2 & \omega_3 \end{bmatrix} \begin{bmatrix} \omega_1 & \omega_2 \\ 0 & \omega_3 \end{bmatrix} + \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}$$

$$+ \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} H_{11,t-1} & H_{12,t-1} \\ H_{12,t-1} & H_{22,t-1} \end{bmatrix} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} + \begin{bmatrix} \gamma_1 z_{t-1}^2 & \gamma_3 z_{t-1}^2 \\ \gamma_3 z_{t-1}^2 & \gamma_2 z_{t-1}^2 \end{bmatrix} \tag{7}$$

or,

$$\begin{aligned} H_{11t} &= \omega_1^2 + \alpha_1^2 \varepsilon_{1t-1}^2 + \beta_1^2 H_{11t-1} + \gamma_1 z_{t-1}^2 \\ H_{12t} &= H_{21t} = \omega_1\omega_2 + \alpha_1\alpha_2 \varepsilon_{1t-1}\varepsilon_{2t-1} + \beta_1\beta_2 H_{12t-1} + \gamma_3 z_{t-1}^2 \\ H_{22t} &= \omega_2^2 + \omega_3^2 + \alpha_2^2 \varepsilon_{2t-1}^2 + \beta_2^2 H_{22t-1} + \gamma_2 z_{t-1}^2 \end{aligned}$$

In this research project maximum likelihood methodology and the BHHH (Berndt, Hall, Hall, and Hausman) algorithm of Bernd *et al*. (1974) was used in the estimation procedure.[3] The relevant data to use for this econometric model is explained in the next section.

3. Price and Storage Cost Data

The data for the agricultural commodities consists of daily spot and futures prices of corn (CN) and wheat (WC) obtained from futures contracts traded at the Chicago Board of Trade (CBOT). The sample period under analysis is twenty-five years from 01/01/1975 to 01/10/1999 supplied by The Futures Industry Institute (FII). The sample size is 6,243 observations. The data for the interest rates consists of daily 91-day Treasury Bills (TB) obtained from

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[3] Since the model is no longer of the usual linear form, ordinary least squares cannot be used. A technique known as maximum likelihood is, therefore, applied. The method works by finding the most likely values of the parameters given the actual data. In other words, by an optimization procedure the relevant parameters of the model equation (6) and (6') which maximize the log-likelihood function are found. BHHH is an algorithm that uses first derivatives to find optimal values of an objective function. It is a modified version of the well known Newton-Raphson algorithm.

the Federal Reserve System (FED).[4] The sample period was chosen considering that it covers sufficient numbers of years including important agricultural U.S. legislation passages of 1985, 1990 and 1996.

The U.S. Department of Agriculture (USDA) kindly provided storage costs for the agricultural commodities under analysis. These were yearly storage fees that the U.S. Government has paid to commercial warehouses in the U.S. to store corn and wheat. The commercial warehouses which have had agreements with the U.S. Government to store grain hold approximately 93 percent of the U.S. capacity to store grain. The data was from the year 1973 to 2000. These yearly storage fees were divided by four in order to determine the relevant three-month period storage cost. The storage costs estimates were used to calculate the adjusted spread for the aforementioned commodities.

3.1. Data Ttransformation

In order to avoid unrealistic "jumps" when creating a time-series of futures prices from different contracts, following Wei and Leuthold (1998), synthetic futures prices were created. These were calculated by a "roll-over" procedure that is basically an interpolation of futures prices from different maturity futures contracts of each commodity, Herbst *et al*., (1989) and Kavussanos and Visvikis (2005). This procedure creates a constant maturity weighted average futures price based upon the futures prices and the days to maturity of the two near-by-expiration contracts.[5] The formula used to obtain the synthetic futures price is shown in Equation (8).

$$SYN_T = F_j\left[\frac{(T-T_i)}{(T_j-T_i)}\right] + F_i\left[\frac{(T_j-T)}{(T_j-T_i)}\right] \tag{8}$$

Where: SYN_T represents synthetic futures price for delivery at T, F_j is the contract j futures price, F_i is the contract i futures price, T is time in number of days, T_i is the contract i expiration in days remaining, T_j is Contract j expiration in days remaining, where j is $i + 1$, with $T_i \leq T \leq T_j$. The time to expiration of the synthetic futures prices calculated as T is 91 days. This is considered an appropriate time-to-expiration given that a shorter time-to-expiration will give higher expected volatility. This situation is observed in empirical research papers, which have found that volatility in futures prices increases, as a contract gets closer to expiration. A higher expected volatility due to time-to-expiration could have biased the results of this analysis. Thus,

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[4] The web page is http://www.federalreserve.gov/

[5] The futures contracts for the aforementioned agricultural commodities have the following delivery months: March, May, July, September and December.

91-day synthetic futures prices were considered appropriate using this method in order to avoid high volatility estimates due to time-to-expiration causes. In addition this will always allow finding a shorter and longer contract, if necessary. For example for a shorter maturity contract T is 30 (one-month) could be targeted. For a longer maturity contract T is 181 (six – months) could be targeted.

4. Descriptive Statistics and Data Analysis

The sample used in this study consists of 6,243 observations from 2 January 1975 to 1 October 1999. Table 1 shows in the second and fourth columns the spot ($\Delta \ln S_t$) and futures returns ($\Delta \ln F_t$), respectively. Table 1 shows the autocorrelation coefficients of daily returns of spot and futures prices of the agricultural commodities under analysis. The last column shows the daily product of the spot and futures returns of these commodities. The results showed that there was weak evidence of time-varying mean in these commodities given that there were few significant coefficients for both spot and futures returns. However, the squared returns columns showed that for both spot and futures prices there was time-varying variance given that all the coefficients were positive and significant.

These results showed that these commodities had time-varying volatility given that the squared returns at time t are estimates of the variances of the spot and futures returns at time t. The cross product of these returns is the measure of their covariance. In addition, the $Q(12)$ Ljung-Box statistic for twelfth-order serial correlation was statistically significant in all cases with the exception of the spot returns of wheat (first column) in which the statistic was not statistically significant. The null hypothesis tested under this test was that all observed values are *i.i.d.*

Table 2 shows descriptive statistics of daily returns and squared returns of the agricultural commodities under analysis. The product of the spot ($\Delta \ln S_t$) and futures returns ($\Delta \ln F_t$) is also reported. It can be observed that the variance of the futures returns is significantly less than the variance of the spot returns. This difference is statistically significant at the 1% confidence level for each commodity. The F-statistics were 2.9636 and 7.9706 for corn and wheat respectively with 6,241 degrees of freedom. These results are consistent with the Samuelson (1965) theory which states that spot returns are more volatile than futures returns. Lastly, it is also worthwhile mentioning that there is high kurtosis in the returns of the variables.

Table 1

Agricultural Commodities Autocorrelations

Lag	$\Delta \ln S_t$	$(\Delta \ln S_t)^2$	$\Delta \ln F_t$	$(\Delta \ln F_t)^2$	$\Delta \ln S_t \, \Delta \ln F_t$
Corn:					
1	0.030*	0.246*	0.052*	0.253*	0.237*
2	0.002	0.226*	-0.014	0.231*	0.244*
3	0.016*	0.164*	-0.001	0.214*	0.184*
4	0.007	0.170*	0.008	0.172*	0.196*
5	-0.017	0.204*	-0.021	0.223*	0.227*
6	-0.001	0.168*	0.010	0.180*	0.167*
7	0.060*	0.190*	0.064*	0.171*	0.197*
8	0.002	0.166*	0.016	0.191*	0.204*
9	-0.003	0.175*	0.028*	0.182*	0.183*
10	0.011*	0.210*	0.018	0.212*	0.255*
$Q(12)$	33.926*	2,613.1*	56.558*	2,799.0*	3,032.0*
Wheat:					
1	0.007	0.063*	0.022	0.225*	0.124*
2	-0.016	0.078*	-0.046*	0.200*	0.158*
3	-0.003	0.021*	-0.002	0.161*	0.124*
4	0.004	0.030*	0.020	0.162*	0.132*
5	-0.024	0.020*	-0.031*	0.157*	0.118*
6	-0.008	0.070*	-0.002	0.171*	0.098*
7	0.002	0.018	0.024	0.155*	0.129*
8	0.041*	0.033*	0.024	0.167*	0.108*
9	-0.004	0.018	0.015	0.121*	0.077*
10	-0.002	0.025*	0.007	0.140*	0.108*
$Q(12)$	16.916	124.23*	34.652*	1,902.7*	946.59*

The row showing $Q(12)$ is the Ljung-Box statistic for twelfth-order serial correlation, which has a χ^2 distribution with 21 degrees of freedom. The critical value is 21 at the 5% confidence level.

* Indicates that coefficients and Q(12) are statistically significant at the 5% level

Table 2

Agricultural Commodities Descriptive Statistics

Agricultural commodity:	Corn	Wheat
VAR($\Delta \ln S_t$)	2.1 x 10^{-4}	2.6 x 10^{-4}
VAR[$(\Delta \ln S_t)^2$]	2.6 x 10^{-7}	1.1 x 10^{-6}
VAR($\Delta \ln F_t$)	1.4 x 10^{-4}	1.7 x 10^{-4}
VAR[$(\Delta \ln F_t)^2$]	9.2 x 10^{-8}	1.27 x 10^{-7}
VAR[($\Delta \ln S_t \; \Delta \ln F_t$)]	1.1 x 10^{-7}	1.85 x 10^{-4}
Kurtosis ($\Delta \ln S_t$)	7.4201	16.1378
Kurtosis ($\Delta \ln F_t$)	5.3923	5.0159
MEAN ($\Delta \ln S_t$)	-0.0001	-0.0001
MEAN ($\Delta \ln F_t$)	-8.79 x 10^{-5}	-8.77 x 10^{-5}

VAR(X) gives the sample variance of the (X) series. MEAN(X) gives the sample mean of the (X) series. KURTOSIS(X) is the coefficient of kurtosis for the variable (X).

Table 3 presents the correlations between the daily returns of spot and futures prices[6] and the daily lagged-adjusted-squared-spread (z^2_{t-1}) for the agricultural commodities under study. CORR(*X,Y*) gives the correlation between the variable *X* and *Y* and partial correlation PCORR($\Delta \ln F_t \, \Delta \ln S_t$, z^2_{t-1}) of the daily product of spot and futures returns and the daily lagged-adjusted-squared-spread (holding the lagged-spot squared and futures squared returns constant) is also reported. It can be observed in Table 3 that the correlation coefficients between the spot and futures squared returns and the lagged-adjusted-squared-spread are positive. This is consistent with the hypothesis that spot and futures returns become more variable when the spread widens.

Table 3
Spread-Squared Return Correlations

	CORN	WHEAT
CORR[$(\Delta \ln S_t)^2$, z^2_{t-1}]	0.1669	0.1137
CORR[$(\Delta \ln F_t)^2$, z^2_{t-1}]	0.1071	0.0209
CORR($\Delta \ln F_t \, \Delta \ln S_t$, z^2_{t-1})	0.1193	-0.0314
PCORR($\Delta \ln F_t \, \Delta \ln S_t$, z^2_{t-1})	-0.0665	-0.0431

All correlation coefficients are statistically significant at 1% confidence level.

In addition, the correlation coefficients between the lagged-adjusted-squared-spread are higher with the spot prices squared returns compared to the futures prices squared returns. This is consistent with economic theory and empirical evidence that have shown that current supply conditions of a commodity has higher impact on spot rather than futures volatilities. Furthermore the partial correlation coefficients between the daily product of spot and futures returns and the lagged-adjusted-squared-spread are negative for both commodities which is consistent with the theory of storage implication that the correlation between the spot and the futures prices decreases as the spread widens. The research papers of French (1986), Fama and French (1987, 1988), Ng and Pirrong (1994) and Susmel and Thompson (1997) provided evidence that is consistent with this implication of the theory of storage.

[6]The spot and futures returns are as described in Table 1.

Table 4 presents unit root tests using the conventional Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. The procedures to test for unit root in the former test are detailed in Dickey and Fuller (1979). In this research paper the augmented form of this aforementioned test (ADF) is performed using logged levels of the variables following the procedure described in Hamilton (1994). The latter test is a non-parametric test, which relaxes the assumption that the errors must be Gaussian white noise (*i.e. i.i.d.)* and normally distributed; and allows for serial correlation and heteroscedasticity, which are commonly observed in spot prices of agricultural commodities, Wei and Leuthold (1998). In the first row, the estimated coefficients are presented, in the second row are the standard errors in parenthesis, the third row shows t-statistics, and the fourth row presents the statistic of the PP test. There are two types of ADF test performed: Model (1) including a constant (T is 0), and Model (2) including a constant and a linear trend. According to the results it can be observed that spot and futures prices are stationary for both commodities under analysis. The ADF-statistic (higher statistic in five lag-ADF regressions) and the PP statistic show that the relevant coefficients are statistically significant at the 5% level *i.e.,* did reject the relevant Mackinnon (1991) critical values of the null hypothesis of a unit root. It is worthwhile mentioning that the adjusted spread (z^2_{t-1}) is stationary for both agricultural commodities as well. The ADF and PP-statistics are statistically significant at the 1% level rejecting the relevant Mackinnon critical values of the null hypothesis of a unit root. These results show the mean reverting characteristic of the interest and storage adjusted spread. The aforementioned characteristic is consistent with the theory.

Table 4
ADF and PP Test Statistics and details about the Estimated Coefficients

	CORN			WHEAT		
ADF TEST	Spot	Futures	Adjusted Spread (z_t)	Spot	Futures	Adjusted Spread (z_t)
(1) γ	–0.0023 (0.0009) -2.6816*	–0.0023 (0.0008) -2.7753*	–0.0105 (0.0018) -5.9363**	–0.0034 (0.0011) -3.0817**	–0.0027 (0.0009) -2.8174*	–0.0096 (0.0018) -5.3906**
(2) γ	–0.0023 (0.0009) -2.7209	–0.0023 (0.0008) -2.7747	–0.0109 (0.0018) -6.0877**	–0.0033 (0.0011) -3.0894	–0.0027 (0.0009) -2.8172	–0.0097 (0.0018) -5.4383**
β	–5.08 x 10^{-8} 1.03 x 10^{-7} -0.4946	-1.44 x 10^{-8} 8.56 x 10^{-8} -0.1679	3.13 x 10^{-7} 2.22 x 10^{-7} 1.4115	-3.28 x 10^{-8} 1.16 x 10^{-7} -0.2832	–1.44 x 10^{-9} 9.43 x 10^{-8} -0.0152	2.22 x 10^{-7} 2.80 x 10^{-7} 0.7927
PP TEST						
(1) γ	–0.0022 (0.0009) -2.6464	–0.0023 (0.0008) -2.7688	–0.0088 (0.0017) -5.0375	–0.0035 (0.0011) -3.2702	–0.0028 (0.0009) -2.9917	–0.0095 (0.0018) -5.3417
t_{pp}	-2.8056*	-2.9404**	-5.6785**	-3.2225**	–2.9814**	-5.4571**
(2) γ	–0.0023 (0.0009) -2.6798	–0.0023 (0.0008) -2.9351	–0.0092 (0.0018) -5.1824	–0.0035 (0.0011) -3.2761	–0.0028 (0.0009) -2.9904	–0.0096 (0.0018) -5.3894
β	–4.35 x 10^{-8} 1.03 x 10^{-7} -0.4236	-8.86 x 10^{-9} 8.56 x 10^{-8} -0.1034	-2.90 x 10^{-7} -2.23 x 10^{-7} -1.3002	-2.65 x 10^{-8} 1.16 x 10^{-7} - 0.2294	5.12 x 10^{-9} 9.44 x 10^{-8} 0.0543	2.23 x 10^{-7} 2.82 x 10^{-7} 0.7907
t_{pp}	-2.8408	-2.9351	-5.8271**	-3.2284*	-2.9801	-5.5053**

(1) Including a constant (T = 0), (2) including a constant and a linear trend.
*** Coefficients significant at the 1% level, ** coefficients significant at the 5% level.
* Coefficients significant at the 10% level. Critical values MacKinnon (1991) in the range of –2.8625 and –2.5673 at the 5% and 10% level respectively for the case of no trend and –3.4132 and -3.1283 at the same significant level for the case of including a trend. The ADF test was carried out with five lags. The PP test truncation lag is ten as suggested by Newey and West (1994).

5. Estimated Coefficients and Analysis of the Results

For the estimation of the coefficients the BEKK(1,1) specification was chosen given that this parsimonious specification gave the smallest values using Akaike Information Criteria[7] (AIC) when it was compared with other higher-order specifications. The details of the different orders of the BEKK

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[7] The AIC is obtained with the following formula: $\frac{-2l}{n}+\frac{2k}{n}$. Where l is the value of the log likelihood function using the k estimated parameters, k is the number of estimated parameters and n is the number of observations.

model are presented in Table 5. As it can be observed in Table 5 the BEKK(1,1) specification gave the smallest AIC values for both commodities.

Table 5
AIC for the BEKK(1,1) Model and Higher Order Specifications

Model Specification	CORN AIC	WHEAT AIC
BEKK(1, 1)	-13.4042*	-12.7841*
BEKK(1, 2)	-13.3747	-12.7261
BEKK(2, 1)	-13.0482	-11.6806
BEKK(2, 2)	-13.0192	-11.6245

* Represents the smallest value.

The results of the Diagonal BEKK(1,1) model without including the adjusted spread presented in Equation (6) can be observed in Table 6. The results of the Diagonal BEKK model including the adjusted spread presented in Equation (6') can be observed in Table 7. It can be observed in Table 7 that with the exception of γ_3 for wheat, the signs of the γ coefficients are positive and statistically significant. These results are consistent with the theory of storage implication that an increase in the basis (adjusted-spread) is positively related to spot and futures prices variability. However it is worthwhile mentioning that the magnitude of the coefficients is not remarkably high.

The sum of the estimated coefficients α_i and β_i is positive and statistically significant for both commodities as it was expected. The sums of both coefficients α_i^2 and β_i^2 are less than one which satisfies a condition in ARCH modelling that their sum must be less than or equal to one. In other words, the volatility of the series is not explosive for any of the commodities. It is worthwhile mentioning that the α_i^2 and β_i^2 coefficients are higher for corn than wheat respectively. On the one hand, this shows that the volatilities of the spot and futures returns have been higher for corn than for wheat. On the other hand the cross-equation covariance influences (β's) have been higher for wheat. An intuition behind these results has to do with the different harvesting seasons for both commodities. Speculators may have been more actively trading in corn compared to wheat considering that most of the harvest of corn in the U.S is after the second half of the year, *i.e.,* July to December. Moreover, wheat is harvested three times during the year thus; this may explain the significant differences of price return fluctuations between both commodities.

Table 6

Estimates of the BEKK(1, 1) Model Excluding the Adjusted Spread

Underlying coefficient	Corn	Wheat
ω (1)	0.0026	0.0026
	(7.55 x 10^{-5})**	(8.13 x 10^{-5})**
	35.1479	*32.4321*
ω (2)	0.0015	0.0018
	(5.76 x 10^{-5})**	(7.79 x 10^{-5})**
	25.8644	*23.0545*
ω (3)	0.0009	0.0012
	(4.37 x 10^{-5})**	(4.19 x 10^{-5})**
	20.6998	*29.3319*
α (1)	0.3491	0.3339
	(0.0048)**	(0.0028)**
	72.7947	*118.2332*
α (2)	0.2997	0.2958
	(0.0053)**	(0.0042)**
	55.9395	*70.3053*
β (1)	0.9205	0.9358
	(0.0019)**	(0.0011)**
	481.3929	*848.7096*
β (2)	0.9444	0.9446
	(0.0017)**	(0.0016)**
	545.1601	*599.5683*
L	41,729.42	39,688.57
AIC	-13.3698	-12.7157
N	6,243	6,243

Standard errors are shown in brackets. ** Indicates the coefficient is statistically significant at the 5% confidence level; * indicates the coefficient is statistically significant at the 10% confidence level. Italics show the *z*-statistic. *L* represents Log-likelihood estimate. *AIC* is Akaike Information Criterion. *N* is sample size.

Table 7

Estimates of the BEKK(1, 1) Model Including the Adjusted Spread

Underlying coefficient	Corn	Wheat
ω (1)	0.0028	0.0032
	(7.92 x 10^{-5})**	(9.96 x 10^{-5})**
	36.1465	*32.6192*
ω (2)	0.0019	0.0023
	(6.38 x 10^{-5})**	(7.97 x 10^{-5})**
	30.0891	*29.2464*
ω (3)	0.0009	0.0011
	(4.64 x 10^{-5})**	(0.0002)**
	20.8035	*18.3867*
α (1)	0.3413	0.3061
	(0.0056)**	(0.0043)**
	61.2128	*71.0171*
α (2)	0.3078	0.2769
	(0.0059)**	(0.0046)**
	51.6586	*59.1772*
β (1)	0.9099	0.9284
	(0.0025)**	(0.0019)**
	363.8831	*475.2325*
β (2)	0.9316	0.9401
	(0.0025)**	(0.0019)**
	368.1771	*505.3481*
γ (1)	8.23 x 10^{-5}	2.78 x 10^{-5}
	(9.39 x 10^{-6})**	(3.00 x 10^{-6})**
	8.7579	*9.2889*
γ (2)	1.35 x 10^{-5}	1.55 x 10^{-6}
	(3.90 x 10^{-6})**	(9.58 x 10^{-7})*
	3.4508	*1.6162*
γ (3)	1.58 x 10^{-5}	-4.17 x 10^{-7}
	(5.53 x 10^{-6})**	(1.55 x 10^{-6})
	2.8544	*-0.2681*
L	41,839.88	39,904.32
AIC	-13.4042	-12.7841
N	6,243	6,243

Standard errors are shown in brackets. ** Indicates the coefficient is statistically significant at the 5% confidence level; * indicates the coefficient is statistically significant at the 10% confidence level. Italics show the *z*-statistic. *L* represents Log-likelihood estimate. *AIC* is Akaike Information Criterion. *N* is sample size.

5.1. Likelihood Ratio Tests

Likelihood ratio tests[8] (LR) were performed to test which model specification is more desirable to use whether excluding (Table 6) or including (Table 7) the adjusted spread. The model specifications including the adjusted spread (Table 7) had higher loglikelihoods compared to the specifications which excluded the adjusted spread (Table 6). Using chi-square values (χ^2), under the null hypothesis of the model excluding the adjusted spread, it was possible to observe that the null hypothesis was rejected in favor of the specification-model including the adjusted spread. The LR statistic for corn was 220.92 which clearly rejected the null hypothesis at the 5% level using three degrees of freedom (d.f.). For the case of wheat the LR statistic was 431.5 which again clearly rejects the null hypothesis at the 5% level using three degree of freedom. The critical value of $\chi^2_{0.05}$ is 7.81.

It is worthwhile mentioning that in the likelihood ratio tests is assumed that the residuals are conditionally normally distributed. However in this case there is evidence that the residuals are not normally distributed in Equation (6) due to the excess kurtosis. The possibility that the residuals are not normally distributed could be considered a shortcoming on the aforementioned likelihood ratio tests given that they may be not fully reliable. Nonetheless in order to have consistent and reliable autoregressive conditional heteroscedasticity (ARCH) parameter estimates the procedure to estimate the quasi-maximum likelihood estimates, covariances and standard errors was performed following the Bollerslev and Wooldridge (1992) methodology. This method ensures consistent parameter estimates and robust standard errors.

5.2. Testing for the Samuelson Effect

Considering the model explained above it is possible to test for the Samuelson hypothesis that the spot volatility is higher than the futures volatility. Using the fitted values for the futures return variabilities and the spot return variabilities estimated in the BEKK model it follows according to this hypothesis H_{22t}/H_{11t} less than one 1. In addition this theory predicts that the spot-return volatility must be larger than the futures-return volatility as the market becomes more inverted, *i.e.* Z_{t-1} increases. Therefore, it will be tested that the correlation between H_{22t}/H_{11t} and Z_{t-1} must be negative. Table 8 presents the Samuelson effect test. The spot return volatility coefficients

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[8] The formula applied for the LR was $-2(L_1 - L_2)$. Where L_1 is the Log-likelihood estimate of the model excluding the adjusted spread and L_2 is the Log-likelihood estimate of the model including the adjusted spread. The critical value was $\chi^2_{0.05}$ = 7.81 with d.f. = 3.

are higher for spot returns than for futures returns for both commodities. In addition the spot return volatilities increase relative to the futures return volatilities when the market becomes inverted (when the adjusted spread increases), hence the correlations between H_{22t}/H_{11t} and Z_{t-1} are negative as this theory predicts. Thus, these results are consistent with the aforementioned theory.

Table 8
Samuelson Effect Test

Commodity	Average futures volatility / Average spot volatility	Correlation between H_{22t}/H_{11t} and z_{t-1}
CORN	0.6788	-0.2767
WHEAT	0.6423	-0.4087

This is analyzed by calculating the coefficient of the average fitted values for the futures returns volatilities divided by the average fitted values of the spot returns volatilities. The fitted values were estimated with the BEKK(1, 1) model.

5.3. Lagged Adjusted Spread and the Price Return Volatilities

Figures 1 and 2 depict the series z^2_{t-1} and the variability of the spot and futures returns for both commodities. The variable z^2_{t-1} is multiplied by –0.0005 in order to compare the series in the same scale. In Figures 1 and 2 *zcn(-1)^2**-0.0005 and *zwc(-1)^2**-0.0005 represent the z^2_{t-1} term for corn and wheat respectively multiplied by -0.0005; (var spot) and (var futures) represent the variance for spot and futures returns respectively for both commodities. It can be observed in both figures that there is a clear relationship between the lagged-adjusted-spread and the variability of the spot and futures returns for both commodities. It can be observed that as the adjusted spread widens the variability of both spot returns and futures returns increases significantly for both commodities. It is worthwhile mentioning that the increase in the variability of spot returns is higher than the increase in the variability of futures returns for both commodities.

Figure 1
Corn Lagged-Squared-Adjusted-Spread and Spot-Futures Price Return Volatilities

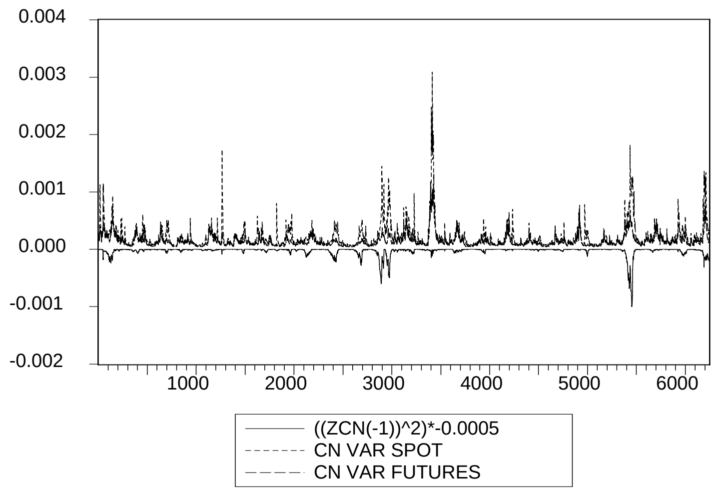
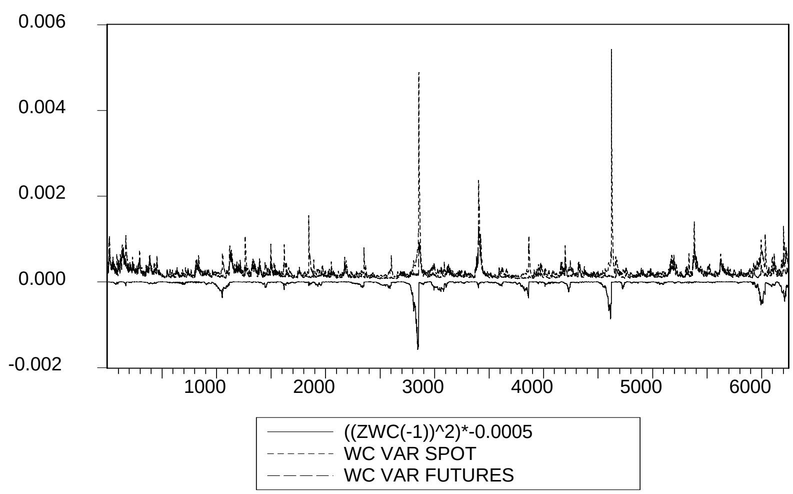


Figure 2
Wheat Lagged-Squared-Adjusted-Spread and Spot-Futures Price Return Volatilities



Conclusions

The results presented in this paper show that the variabilities of the spot and futures returns were statistically significant and positively related to the lagged-adjusted-spread as the theory of storage predicts. In addition it can be clearly observed in Figures 1 and 2 that when there was an increase in the lagged-adjusted-spread (widens) there was also an increase in the spot and futures returns volatilities for both commodities. It is worthwhile emphasizing that both figures show that the variability of the spot returns was higher than the variability of the futures returns when the aforementioned spread widens.

Lastly, the results presented were consistent with the Samuelson hypothesis that the spot prices are more volatile than futures prices. In addition, the correlations between the ratio of the futures and spot return volatilities and the lagged-adjusted-spread were negative as this theory predicts. In a few words, the results estimated using the restricted version of the BEKK model for these commodities were in line with both theories.

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