*Ensayos–Volumen XXVIII, núm.1, mayo 2009, pp. 61-80*

# "Last-chance" sales: what makes them credible?

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Fecha de recepción: 16 XII 2008 Fecha de aceptación: 16 IV 2009

**Abstract**

This paper analyzes the firms' standard practice of announcing clearance or "last-chance" sales, namely advertising that a particular product is not going to be available in the market anymore. In the context of a two-period signaling game, prices and advertising decisions of firms are analyzed. Then, the set of separating and pooling equilibria is characterized, so that the above usual advertising techniques can be better understood as equilibria of this model for certain parameter values. In particular, this paper shows that, when the firm which continues in the business knows that few of their current customers will come back in future periods, the set of separating equilibria shrinks. That is, fewer future prospects induce all types of firms to compete for current consumers, leading to pooling equilibria in which all firms announce a "last-chance" sale, even if some of them know they will remain in the industry next period.

***Keywords***: signaling, advertising, separating equilibria, information transmission.

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## Resumen

Este artículo analiza la práctica de múltiples firmas respecto de anunciar grandes liquidaciones o "últimas rebajas"; en particular analiza el anuncio de que un producto concreto no estará disponible en el futuro. En el contexto de un juego de señalización de dos etapas, analizamos las decisiones de las empresas con respecto a los precios de sus productos y sus gastos en publicidad que anuncian liquidaciones. Esto permite caracterizar el conjunto de equilibrios separadores y agrupadores, lo que nos deja en condición de ofrecer una explicación de las anteriores técnicas publicitarias sobre grandes liquidaciones en el contexto de los equilibrios de nuestro modelo, para ciertos parámetros. En definitiva, este artículo demuestra que cuando la firma que continúa produciendo sabe que pocos de sus clientes volverán a realizar compras en el futuro, el conjunto de equilibrios separadores se contrae. En otras palabras, peores perspectivas de futuro inducen a todo tipo de empresas a competir por los clientes actuales, llevándolas hacia un equilibrio agrupador en el que todas las firmas anuncian grandes liquidaciones, incluso aquellas que saben que continuarán en el mercado durante el siguiente periodo.



***Palabras clave***: señalización, publicidad, equilibrio separador, transmisión de información.
***Clasificación JEL***: L12, D82.


## Introduction

Why do so many firms constantly insist that a certain store, or the entire company itself, is going out of business? Can we believe it, or should we assume that all types of firms have incentives to use this advertising technique? This paper analyzes these questions in a two-period signaling model. In particular, firms play here the role of the informed participant of the market, since they know whether firm will exit the market in the next period. After observing the firm's advertising and prices, consumers update their beliefs about the producer's probabilities of actually leaving the industry, and then decide whether to buy today, or wait until the next period. Specifically, we find what strategies and parameter values can support either a pooling equilibrium -in which all types of firms announce their imminent closing and set low (clearance) prices- or a separating equilibrium -where only certain announcements are believed. In addition, in order to restrict the set of admissible equilibria, Cho and Kreps' (1987) intuitive criterion is used, so that only equilibria in which consumers sustain relatively "reasonable" beliefs are considered.

Finally, different comparative statics analysis are carried out, which can specially help us to understand why, for example, this type of advertising campaigns have massively expanded in recent years, or why we normally observe more of such advertising in cities than in small towns. In particular, we can explain the above phenomena by the fact that an increase in the probability that a customer returns to a store shrinks the set of separating equilibria, i.e., the set of admissible price-advertising combinations that the exiting firm can use. Particularly, the probability that a particular customer returns to a given store, is smaller in a big city than in a small town, and it has probably decreased in recent decades as the number of stores -as well as the introduction of internet shopping- became more important. In turn, this smaller probability of having returning customers induces the firm that stays in the market to only concentrate on the first period potential customers, trying to convince them through prices and advertising that its product is not going to be available in future periods. Therefore, firms' tendency towards a pooling equilibrium in which they all try to persuade consumers with “last-chance” sale advertising would be supported when the probability that customers return to the store is sufficiently low.

The paper is organized as follows. The next section considers the literature on signaling models, with special attention to quality signaling and the literature on multi-stage signaling games. Section 2 describes the preferences of consumers and firms, as well as the precise time structure in which this "last-chance-sale" signaling model evolves. Then, section 3 analyzes the conditions under which a set of price-advertising combinations can be supported as a separating (or pooling) equilibrium of the signaling game. In this section, we also included some comparative statics results, which might shed some light on the main questions addressed in this paper. Afterwards, section 4 refines the set of equilibria found in previous sections by eliminating those equilibria sustained by “unreasonable” off-the-equilibrium beliefs. Finally, we summarize the main results of this paper and comment about its further extensions.

## 1. Related literature

The literature on signaling models, initiated by Spence (1973), has been used to explain a wide array of economic -but also political- situations, from limit pricing [Battacharya (1979) and Kose and Williams (1985)], to dividend policy [Milgrom and Roberts (1982)] and warranties [Gal-Or (1989)]. Interestingly, these models have also produced different explanations about uninformative advertising, i.e., the type of advertising where the information provided by the firm is unverifiable unless the consumer actually buys the good. In particular Nelson (1974) initially suggested the possibility that this kind of advertising -such as the one conducted by a company which introduces a new good in the market- could work as a signaling device that firms use to distinguish the quality of their products. Specifically, bad quality firms would not find convenient to imitate such an advertising campaign since it is costly, and if it had any effect on the consumers' buying decision, this effect would vanish in the next period, as soon as buyers realize the bad quality of their product.

The formalization of these ideas into a signaling model by Milgrom and Roberts (1986) provided the methodology for further research in this area. For instance, from a theoretical approach, Bagwell and Riordan (1991) analyze the signaling effect of highly stable versus declining prices as the number of customers of a given good increases, and Linnemer (2002) considers a similar model in which advertising is also included. In addition, Milgrom and Roberts' (1986) theoretical contribution triggered some empirical literature, such as Horstmann and MacDonald (2003), which checks the role of advertising as a signal of product quality in the compact disc player industry. Interestingly, many of these models consider multidimensional signaling, since firms are able to reveal their quality to the customers through prices and advertising. Additionally, they usually assume that all those consumers who bought the high-quality product buy it again in the future. Finally, Lazear (1986) analyzes clearance sales by showing how firms choose a declining path of prices for the good (such as fashionable goods).[3] Our model also examines firms' price setting behavior, but complements it by introducing their optimal advertising strategies.

This paper models a "last-chance" sale with a similar methodology to the above literature: firms can use both prices and advertising announcing their clearance situation to signal their type. Furthermore, customers can decide whether to return to the firm which is still in business, although this firm's products are perceived as not exclusive by consumers, since they can easily be found in the market along different periods of time. This fact reduces, hence, the firm's incentives to reveal its type when it is continuing its business. In addition, we allow for only a proportion of these customers to come back to this firm in the future. As this paper shows, the particular pattern of returning customers greatly conditions the pricing and advertising strategy of the firm which remains in the industry, which in turn helps in the understanding of the widespread use of such "last-chance" sale advertising campaigns.

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[3] Extensions of this model include Epstein (1998) for applications to multiple goods, and Nocke and Peitz (2007) for extensions to firms with capacity constraints.

## 2. Model

Let us consider a signaling game in which consumers buy a (non-durable) consumption good from a firm. Initially, the firm privately observes its “type” $\theta=\{L,H\}$ representing a low or high amount of product in the firm stocks. The low-stock firm represents here the seller (or product) which exits the industry in the next period, and the high-stock producer denotes the firm who keeps its business. Information about the level of stocks, $\theta$, is not observed by consumers. In particular, the time structure of this game is the following:

1. Nature selects low or high stocks for the firm, with probability $q$ and $1-q$ respectively. This information is revealed to the firm, but not to consumers. The stocks situation of a firm is assumed to be persistent over both periods of time.

2. The firm sets a price for its products $p \in \mathbb{R}_+$, and spends an amount $A \in \mathbb{R}_+$ on specific advertising which informs that this is the last chance of acquiring a certain product.

3. Observing the price-advertising combination ($p$, $A$) consumers update their beliefs about the probability that this combination comes from a low or a high stock firm.

4. Afterwards consumers choose whether they buy today at the current price, or they wait until next period. If consumers decide not to buy today, then they are only able to buy tomorrow if the true type of the firm is $H$.

5. If consumers decided not to buy in the previous period and the firm has high stocks, then the firm is called to set a second-period price, $p_2$. If the firm had low stocks, then it exits the industry after the first period. In this period firms can set prices, but they cannot spend on advertising.

6. Only a portion $\alpha$ of consumers returns to the same store in the second period. After observing second-period prices, $p_2$ and having perfect recall of the actions they observed in the previous period, consumers decide whether to buy the product, given their updated beliefs about the firm's true type.

Specifically, every consumer can only buy in the second period from a firm with (persistent) high stocks. Otherwise -if the firm has low stocks- we assume that it exits the industry after the first period. Let $\pi(p,\theta,s)$ denote the payoff of a firm with true type $\theta=\{L,H\}$, which is perceived by consumers as a $s$-type firm, where $s=\{L,H\}$, and charges a price $p$ in that particular period.[4] For example, $\pi(p,H,L)$ represents the payoff for a high-stock firm which is perceived by customers as a low stock firm, i.e., consumers believe that the firm will exit. As this example emphasizes, $s$ does not necessarily coincide with $\theta$. Firm's profits are increasing in the price charged by the firm, $p$, no fixed costs are assumed, and no assumptions about the functional form of $\pi(p,\theta,s)$ are made[5]. Furthermore, let $\pi(p_{HH},H,H)$ represent the maximum profits that a high-stock firm perceived as such by consumers- can attain by selecting the optimal price $p_{HH}$. We use a similar notation for the maximum profit level of any other combination of true and perceived firm's type.

In addition, we assume that a firm's operating costs are lower when it is closing its business (low stocks) than when it still continues in the industry (high stocks). This assumption can be rationalized by considering that the opportunity cost of not selling today is higher for the high-stock than for the low-stock firm. Indeed, the high-stock seller is able to sell in the next period at a price above marginal cost. However, the low-stock seller cannot sell its products in the market anymore, so any remaining stocks from the first period must be sold at a price equal to its marginal cost during the second period, when the firm liquidates its assets. Hence, for any given price and consumers' beliefs, the low-stock firm's profits during the first period are higher than those of the high-stock firm, i.e., $\pi(p,L,s)>\pi(p,H,s)$. Finally, note that this payoffs are only considering revenues and costs for a single period of time (either $t$=1 or $t$=2) and do not take into account advertising expenditures, $A$.

Let the payoff of the representative consumers be $U(p,\theta)=\theta-p$ from consuming one unit of the good (consumers demand either one or zero units) when the price of the good is $p$, and the stock level of the firm is $\theta=\{L,H\}$ where $L,H\in\mathbb{R}_+$. In particular, we assume that consumers prefer to consume goods from firms that are really leaving the industry. That is, at a given price $p$, $U(p,L)>U(p,H)$. This preference for products which will become scarce in the future can be understood in terms of a "snob effect", or in a more general

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[4] Note that $\pi(p,\theta,s)$ represents the case in which the firm is the monopolist in the market, as well as other cases in which the product sold by the firm is sufficiently differentiated from that of other similar firms.

[5] Some additional conditions about $\pi(p,\theta,s)$ are only needed in section 4.

setting, because of a consumer's intention to price arbitrage along time[6]. Additionally, consumer's preferences are decreasing in the price paid for the good, $p$.

## 3. Analysis of the sequential equilibria

In this section we find the set of price-advertising combinations $(p,A)$ which can be sent by the low stock firm, which is close to exiting the industry in order to fully reveal its stock conditions to the customers (separating equilibria). That is, we look for $(p,A)$ pairs such that: (1) the low-stock firm prefers to send rather than being perceived as a high-stock producer; and (2) the high-stock type of firm do not find convenient to send -does not want to mimic the low-stock firm. The first of these incentive compatibility conditions is given by

$$\pi(p,L,L)-A>\pi(p_{LH},L,H) \tag{1}$$

where $p$ denotes a price level that, in addition to some “last-chance-sale” advertising, allows the low-stock firm to distinguish itself from the high-stock seller. Note that in the above inequality no second period payoffs are included, since the firm exits the industry after the first period, and it cannot obtain any additional profits in the future, regardless of the beliefs that consumers may have about its actual stock conditions. Similarly, the second incentive compatibility condition, about the high stock producer, determines

$$\pi(p_{HH},H,H)+\alpha\pi_2(p_2,H,H)>\pi(p,H,L)-A \tag{2}$$

Intuitively, the left hand side represents the high stock firm's equilibrium profits in a separating sequential equilibrium in which the firm is perceived as a high stock firm by consumers. As this expression indicates, the high stock firm payoffs are twofold: On one hand, its first period profits from being perceived as a high stock firm (which are maximized when the firm sets a price level of $p_{HH}$). On the other hand, and since consumers identify its high stock condition, the firm can also sell in the future with a probability $\alpha$. This probability can be interpreted as the proportion of customers who return to the store in the next period. If this is the case, the firm sets a profit

[6] Different assumptions are valid regarding consumers' preferences. For example, many consumers may prefer to not buy during massive clearance sales in order to avoid using the same clothes, goods, etc. as the middle class, probably overrepresented in the population of buyers at these clearance sales. In this case, the “snob buyer” would prefer the high-stock firm products, so that he can keep some status from his purchases. The implications of such behavioral pattern are, however, beyond the scope of this paper.

maximizing price of $p_2$. The particular value of $p_2$ that the firm sets must be $p_2=H$, since now the consumers perfectly infer the firm's stock condition from the fact that it is still operating.[7]

**Lemma 1.** *The high-stock firm's profit maximizing price in the second period is* $p_2=H$.

Regarding the right hand side of (2), it denotes the utility of the high-stock firm from setting a price $p$ that, together with some "last-chance-sale" advertising, allows this high-stock firm to be perceived by customers as a low-stock producer.

In addition to the above two incentive compatibility conditions, individual rationality constraints must also be considered for both types of firms, since otherwise they would not voluntary participate in this market. Specifically, we need that any firm's payoffs from this separating sequential equilibrium are higher than their reservation utility from not participating, which can be normalized to zero, as the following inequalities indicate.

$$\pi(p,L,L)-A>0$$

$$\pi(p_{HH},H,H)+\alpha\pi_2(p_2,H,H)>0$$

These conditions are, however, derived from expressions (1) and (2) respectively. Indeed, if a firm's profits from not truthfully revealing its type (right hand side) are negative, a firm can set an extremely high price, which guarantees no sales and zero profits. That is, $p_{LH}$ in expression (1) (or respectively $p$ in expression 2) can be made as high as needed, so that the right-hand sides of (1) and (2) are never negative. Hence, thereafter we consider conditions (1) and (2) as the only requirements for a separating (fully revealing) sequential equilibrium. Combining both conditions we obtain a range of values for the advertising expenditures in any separating equilibrium.

$$\pi(p,L,L)-\pi(p_{LH},L,H)>A>\pi(p,H,L)-\pi(p_{HH},H,H)-\alpha\pi_2(p_2,H,H)$$

In order to identify the set of price-advertising pairs $(p,A)$ that can be supported as a separating equilibrium -i.e., which satisfy conditions (1) and

[7] Note that this must be both the original and the final stock condition of the firm in this two-period game, since we assumed that, for simplicity, the firm's stocks are persistent over both time periods.

(2) above- it is useful to represent the isobenefit curves of the low and high-stock firms. In particular, we are interested in the locus of (*p,A*) combinations for which these conditions bind. Formally, we can then define the low-stock firm isobenefit curve by,

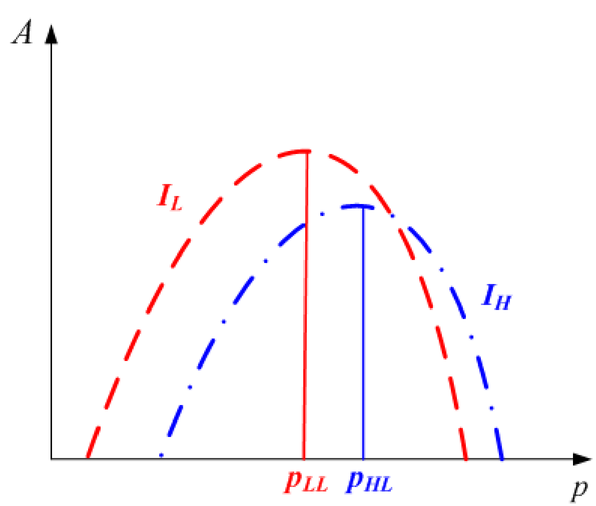
$$I_L \equiv \{(p,A) \in \mathbb{R}^2 \,|\pi(p,L,L) - A = \pi(p_{LH},L,H) = \pi(p,H,L)\ \}$$

And, using expression (2) we can similarly characterize the high-stock firm's isobenefit curve

$$I_H \equiv \{(p,A) \in \mathbb{R}^2 \,|\pi(p_{HH},H,H) + \alpha\pi_2(p_2,H,H) = \pi(p,H,L) - A\}$$

Graphically, all those (*p,A*) pairs below the $I_L$ curve represent points for which condition (1) holds strictly. Equivalently, (*p,A*) combinations above $I_H$ satisfy condition (2).

Figure 1




Importantly, both firm's isobenefit curves are concave in *p*. Indeed, for example, for a given level of the low-stock firm's profits, and given that the marginal costs of advertising are constant, the concavity of $I_L$ is satisfied because of the pattern of the marginal utility from increasing prices for a seller. In particular, this marginal utility from a price increase is positive until $p_{LL}$ -intuitively, prices where the corresponding interval of the linear demand curve is inelastic- and beyond $p_{LL}$ the marginal utility from rising prices is negative -elastic intervals of the demand curve.[8] In other words, the marginal ratio of substitution between prices and advertising is positive until $p_{LL}$, but negative thereafter. A similar analysis is applicable for the isoprofit $I_H$.

Additionally, note that lower isobenefit curves -curves closer to the price axis- represent higher profits for either firm. Further, since $\pi(\ )$ do not include the advertising decision of the firm, one can observe that prices like $p_{LL}$ represent the profit maximizing price, given a particular advertising level $A$, for the low-stock firm when being perceived as such by consumers. Indeed, for a fixed level of $A$, the price $p_{LL}$ picks up the lowest isobenefit curve (the isobenefit curve closer to the price axis).

Then, in a separating equilibrium, both types of firms want to reach the lowest possible isobenefit curve which is compatible with revealing its type to the customers. Hence, a low-stock firm chooses any ($p,A$) pair in the area below $I_L$ and above $I_H$. If this were the case, a high-stock firm would find imitation profitable. Therefore, price-advertising pairs below $I_L$ and above $I_H$ offer the low-stock producer a greater profit than it could get by hiding information about its stocks. In addition, such ($p,A$) combinations offer a high-stock producer lower payoffs than it could obtain by setting advertising to zero and revealing itself as a high-stock firm. Therefore, the area below $I_L$ and above $I_H$ denotes all price-advertising combinations which can be supported as a separating equilibrium of this signaling game. Let us now analyze some comparative statics derived from the above incentive compatibility conditions.

### 3.1. Comparative statics

Let us analyze the comparative statics effects of varying second period incentives. In particular, we consider the dynamic incentives of the high-stock producer, and specifically, the trade-off it experiences between selling today and waiting until the next period for future customers. We include proofs in the appendix.

**Lemma 2.** *The set of price-advertising pairs (p,A) that can be supported as a separating sequential equilibrium of the "last-chance-sale" signaling game increases in the proportion of customers, α, who come back to make purchases at the high-stock firm.*

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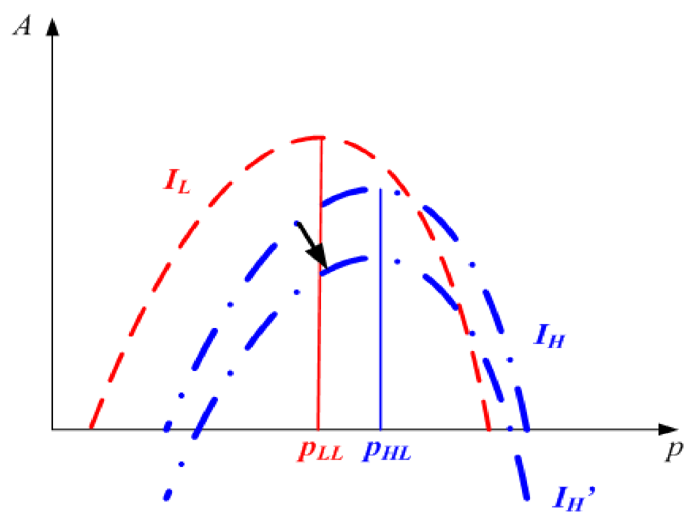
[8] Note that this assumption is applicable to both linear and non-linear demand curves, as long as the price-elasticity of demand varies as a result of price changes.

The intuition of this lemma becomes clear if we look at condition (2), and check the effects of increasing $\alpha$ in the high-stock firm isobenefit curve. As the following figure indicates, the isobenefit curve $I_H$ shifts downwards as $\alpha$ increases (i.e., reaching higher isobenefit levels). That is, as the proportion of customers who might come back to the high-stock firm rises, this producer is more willing to give up some first period profits. Indeed, this firm still obtains some profits, but higher than those it would obtain if it was able to identify its product as a “last-chance-sale” greatly appreciated by the customers. The cost of so doing for the high-stock seller would be to totally lose any second period sales, since the store would be identified as low-stock (having no future customers). This cost is clearly increasing in α, the proportion of customers who come back. Graphically, the set of $(p,A)$ pairs below $I_H$ shrinks, and then the set of $(p,A)$ combinations that can support separating sequential equilibria -area between $I_H$ and $I_L$- expands. Intuitively, the low-stock seller can now distinguish itself from the high-stock one by sending higher prices and smaller advertising expenditures. In other words, increases in α benefit the low-stock producer since now its separating messages are “cheaper.” In contrast, the set of $(p,A)$ pairs that can support separating sequential equilibria shrinks as $\alpha$ is further decreased.

Figure 2
**Effects of an increase in $\alpha$**



The above result can help us understand why, in recent years, so many advertising campaigns have insisted in the fact that their sales are clearance or "last-chance" sales. Indeed, more mobile customers -with an increasing array of traditional and internet shopping centers to choose from- reduce the probability that an individual customer returns to the same store (i.e., decreases $\alpha$). Hence, the set of $(p,A)$ pairs which can support a fully revealing sequential equilibrium shrinks, and induces more firms to play pooling equilibria. In this situation, as commented above, both types of stores -the ones closing their business and those which continue their operations- set prices and advertising levels which try to convince customers about a "last-chance-sale" situation.

The same idea is valid to explain why this type of advertising techniques are more commonly observed in cities than in small towns: the probability that a given customer returns to a particular store is significantly smaller in a big city than in a town, where the number of stores he can choose from is smaller. Summarizing, significant decreases in $\alpha$ increases the high-stock firm incentives to mimic the "last-chance-sale" strategy of the low-stock sellers, producing the recurrent pooling equilibrium.

***Remark 1***. The above reasoning is also applicable to the second period profits. Specifically, note that since $p_2=H$, when the difference between $H$ and the unit cost of this firm is small, then the profits per unit of the high-stock seller decrease, reducing $\pi_2(p_2,H,H)$. The effects of this decrease into the set of separating equilibria are analogous to a reduction in $\alpha$: the set of separating sequential equilibria shrinks.

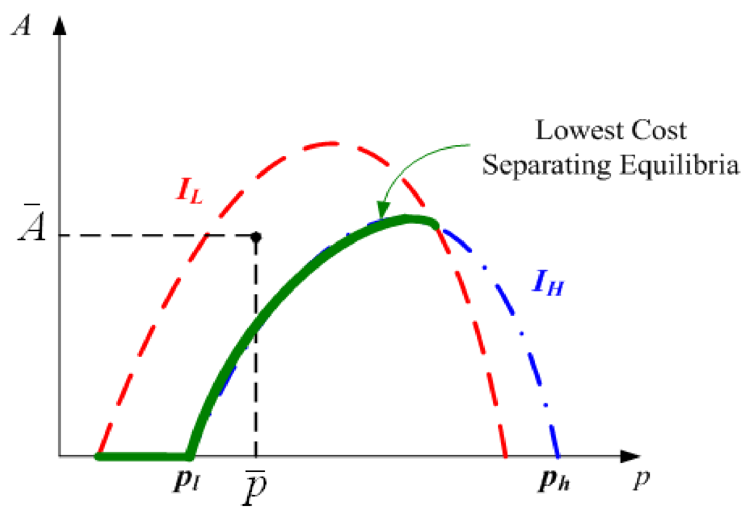
***Remark 2***. Finally, note that all this analysis and results can be extended to the case in which α represents the high-stock seller's discount rate, $\delta$. In this case, a higher discount rate would indicate that the firm assigns a greater importance to future payoffs, and for this reason, it does not want to give them up (something that would happen if it mimics the low-stock seller). On the other hand, when $\delta$ approaches zero the high-stock firm does not assign any relevance to future profits, and tries to mimic the low-stock producer's strategies as much as possible given its technology[9].

[9] Note that even when $\alpha$=0, as commented above -both in the interpretation of $\alpha$ as a proportion of returning customers and as the firm's discount rate- the high-stock seller can find $(p,A)$ pairs which does not want to imitate.

## 4. Equilibrium refinements

Let us now turn to the set of separating sequential equilibria found in the previous section. This set is indeed large and reduces the predictive power of the analysis. In order to overcome this problem, in the next section we apply Cho and Kreps' (1987) intuitive criterion, which greatly reduces the set of separating sequential equilibria by allowing only "reasonable" off-the-equilibrium beliefs. By so doing, we identify the set of Lowest Cost Separating Equilibria (LCSE) of this signaling game. The intuition of this criterion, when applied to this game, is clear: the low-stock firm just selects those (*p,A*) combinations which maximize its profits, provided that these combinations are able to signal its low-stock situation to the consumer. The low-stock firm achieves this in the solid line of the following figure, which exactly specifies those lowest price-advertising combinations among the ones which can be supported as a separating sequential equilibrium of the game.

Figure 3
**Lowest Cost Separating Equilibria**




**Proposition 1.** *From the set of price-advertising pairs (p,A) which can be supported as a separating sequential equilibrium of the game, only those satisfying*

$$\pi(p_{HH},H,H)+\alpha\pi_2(p_2,H,H)=\pi(p,H,L)-A \tag{3}$$

*survive the Cho and Kreps' (1987) intuitive criterion. Let this set of remaining equilibria be denoted as Lowest Cost Separating Equilibria (LCSE).*

In order to formally characterize the set of LCSE, firstly note that a necessary condition for a positive level of advertising, $A$>0, is that (see figure 3),

$$p^* \in (p_l, p_h) \tag{4}$$

This condition is not sufficient, however, for $A$>0. Graphically, we can see that a sufficient condition for positive advertising is that the isobenefit curves of both types of firms must be tangent at a price level $p^T$ such that

$$p^T \in (p_l p_h) \tag{5}$$

Finally, we must check that the above equilibrium indeed maximizes the low-stock firm profits. Indeed, ($p^*,A^*$) maximizes this firm profits as long as

the isobenefit curve $I_L$ is quasiconcave in $p$ (6)

Otherwise ($p^*,A^*$) would be a minimum of this firm profits, and there would be no separating equilibria with positive expenditures on advertising.

**Proposition 2.** *Take the set of LCSE.*

1. *If conditions (4)-(6) hold, then there exists a separating sequential equilibrium with positive advertising. If, in addition, condition (6) holds strictly, then there exists a unique separating equilibrium with positive advertising.*

2. *If either condition (4), (5) or (6) fails, then all separating equilibria involve zero advertising.*

Figure 4(a) illustrates the first point of the above proposition: a LCSE with positive advertising. If the low-stock firm's isobenefit curve is strictly quasiconcave as depicted in the figure, the separating equilibrium is unique. If this condition holds with equality there are, as a consequence, a continuum of ($p,A$) tangency pairs which satisfy conditions (4)-(6). In contrast, if either condition fails, then there does not exist any tangency point within the price interval ($p_l,p_h$). In this case, separating equilibria with no advertising exist, as figure 4(b) indicates. In fact, note that the equilibrium profile for this case prescribes no advertising but a significantly low price, so that the low-stock firm can clearly signal its stock situation if it uses a low enough price.

Figure 4(a)

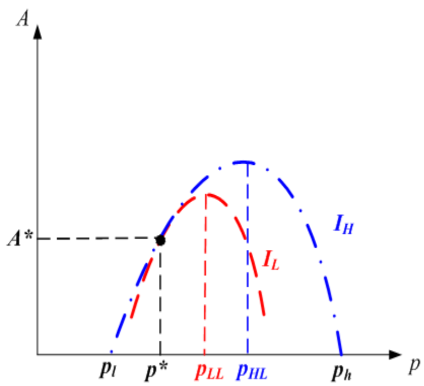
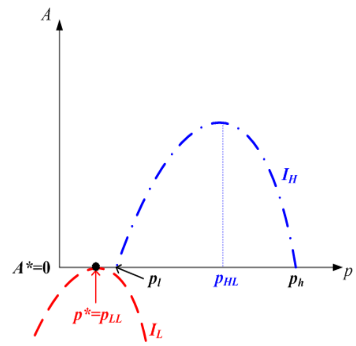



Figure 4(b)




## Conclusions

This paper analyzes a two-period signaling model in which a firm, which either continues or exits the industry, decides what price level to set and how many resources to spend on advertising. These two choice variables, in a separating equilibrium, are used by the firm to signal to its potential customers whether the firm will exit or stay in the market. We identify the set of prices and advertising, $(p,A)$, which support fully revealing separating sequential equilibria. In addition, this set of separating equilibria was restricted to allow only "reasonable" beliefs, applying Cho and Kreps' (1987) intuitive criterion.

The effects of modifying the dynamic incentives for the firm which continues in the market are also considered in a comparative statics analysis. In particular, reducing the proportion of returning customers expands the set of pooling equilibria, inducing all types of firms to announce "last-chance-sales." Similar effects are also identified for the case that second period profits are small, and when the sellers assign a low discount rate to future payoffs.

It would be interesting to extend this two-period signaling game to a repeated signaling game. Indeed, firms and consumers interact in long relationships along time: some firms exit the market in the next time period, while other firms stay in business. In each period consumers and firms would play a signaling game in which consumers updated their beliefs given, not only the current signals from the firm, but also all the past history of play. In this case the assumptions on the stochastic process of the random variable $\theta=\{H,L\}$ over time become very relevant. If $\theta$ is assumed to be persistent along periods, then after the actual realization of θ is known at the end of period one, the continuation game starting at period two becomes a usual repeated game with complete information, as in Kaya (2004).

If, in contrast, $\theta$ is assumed to be independently drawn each period of time, then past histories do not provide relevant information for the consumer that he could use in his belief updating. That is, the signaling game analyzed in this paper is repeated once and again, each period game being an isolated event. Probably, the most interesting repeated signaling model is that in which the stochastic process generating $\theta$ is correlated along time, i.e., the probability of drawing $\theta=H$ in period $t$ depends on the probability of that particular realization in previous periods. Finally, it would also be important to extend the model to contexts where product quality is *not* perfectly observable to customers. In such case, advertising expenditures would be used as a signal of quality *and* stocks.

## Appendix

### *A.1 Proof of Lemma 1*

In the second period of the game, no other firm operates in the market, and consumer´s beliefs when observing any firm are concentrated on high stocks. Then, the firm sets a price $p_2$ such that makes the consumer indifferent between buying and not buying the product, $U(p_2,H)= p_2-H=0$, i.e., $p_2=H$.

### *A.2 Proof of Lemma 2*

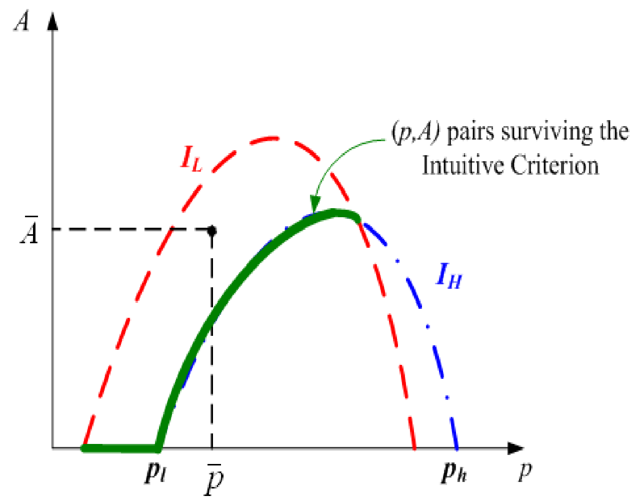
From expression (2) we know that the incentive compatibility condition of the high-stock firm is $\pi(p_{HH},H,H)+\alpha\pi_2(p_2,H,H)>\pi(p,H,L)-A$, which indicates all ($p,A$) pairs above the isoprofit $I_H$.

$$I_H \equiv \{(p,A)\in \mathbb{R}^2 | \pi(p_{HH},H,H)+\alpha\pi_2(p_2,H,H)=\pi(p,H,L,)-A\}$$

For a given increase in α, the set of ($p,A$) pairs above the isoprofit $I_H$ enlarges.

### *A.3 Proof of Proposition 1*

Let us assume that ( $\overline{p}$ , $\overline{A}$ ) is the equilibrium strategy profile of the low-stock firm in a separating sequential equilibrium, as the figure below illustrates.




*First Step:*

Let us first restrict consumer's beliefs when an off-the-equilibrium message is observed (i.e., when a price-advertising pair ( $\hat{p}$ , $\hat{A}$ ) ≠ ( $\overline{p}$ , $\overline{A}$ ) is observed) to only those types of firms from whom this off-the-equilibrium message is never equilibrium dominated. In order to simplify the consideration of different (*p,A*) pairs, let us start from pairs close to the $I_L$ frontier, and then take (*p,A*) combinations below $I_L$, so that the firm's profits are higher (lower isobenefit curves). Formally,

$$\Theta^{**}(\hat{p}, \hat{A}) = \{\theta \in \Theta \mid \pi(p,\theta,s) - A < \max_{s \in S^*(\Theta,\hat{p},\hat{A})} \pi(\hat{p},\theta,s) - A\} = \{L\}$$

Since the *L*-type of firm can benefit from sending different messages (in the area) but the *H*-type of firm does not.

*Second Step:*

Once consumers' beliefs have been restricted to Θ**, we must check whether the equilibrium payoff is smaller than the one associated with the restricted set of beliefs. Indeed

$$\pi(p,L,L)-\overline{A} < \pi(p,L,L)-\hat{A} \text{ for all } (\hat{p},\hat{A}) \neq (\overline{p},\overline{A})$$

Therefore, the initial equilibrium strategy profile $(\overline{p},\overline{A})$ does not survive the intuitive criterion. The same argument can be carried out for any price-advertising pair which strictly belongs to the area representing the set of separating sequential equilibria.

For any pair $(p,A)$ in the boundary of the area representing the set of separating equilibria, however, the intuitive criterion does not eliminate any equilibrium because of holding “unreasonable” beliefs for off-the-equilibrium messages. Indeed, the first step is applicable to all of them just as above. However, the second step does not eliminate any of these $(p,A)$ pairs.

### *A.4 Proof of Proposition 2*

If condition (4) fails, then $p_{LL}<p_l$ or $p_{LL}>p_h$, which implies $A$=0. If condition (5) fails, then the tangency point between isoprofits $I_L$ and $I_H$ occurs at a price level $p^T<p_l$ or $p^T>p_h$, which also implies $A$=0. If condition (6) fails, then the firm could reduce $A$ increasing profits. Then, all conditions (4)-(6) must be satisfied for a LCSE to be supported with positive advertising, $A$>0. In addition, if condition (6) holds strictly, the tangency point between the isoprofits $I_L$ and $I_H$ is unique, and the equilibrium is unique.

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